DOMINATION AND EDGE DOMINATION IN TREES[1]

B. Senthilkumar[†], Y. B. Venkatakrishnan[††], H. Naresh Kumar[†††]

SASTRA Deemed University, Tanjore, Tamilnadu, India

[†]senthilsubramanyan@gmail.com   [††]venkatakrish2@maths.sastra.edu
[†††]nareshhari1403@gmail.com

Abstract: Let $G = (V, E)$ be a simple graph. A set $S \subseteq V$ is a dominating set if every vertex in $V \setminus S$ is adjacent to a vertex in S. The domination number of a graph G, denoted by $\gamma(G)$ is the minimum cardinality of a dominating set of G. A set $D \subseteq E$ is an edge dominating set if every edge in $E \setminus D$ is adjacent to an edge in D. The edge domination number of a graph G, denoted by $\gamma'(G)$ is the minimum cardinality of an edge dominating set of G. We characterize trees with domination number equal to twice edge domination number.

Keywords: Edge dominating set, Dominating set, Trees.

1. Introduction

Domination theory is a well studied topic in graph theory. Depending on the utility in real life application, domination on vertex set and on edge set has been defined. Edge dominating set is used to study the behaviour of telephone switching network [5] built to phone calls from one telephone to another telephone at a time. Edge dominating set is also used in deterministic distributed algorithms in networks with unique node identifier in port numbered network. Dominating set is used to identify the minimum number of servers in an adhoc network. For different dominating parameters, the reader is refered to two excellent books [2, 3].

In domination theory, comparison is made between domination parameters defined on vertex set or domination parameters defined on edge set. There are only a few studies on comparison between domination parameter defined on vertex set with a domination parameter defined on edge set, see [4, 8]. Here, a domination parameter defined on edge set, edge domination, is compared with a domination parameter defined on vertex set, vertex domination and we characterize trees with domination number equal to twice edge domination number.

Let $G = (V, E)$ be a simple connected graph. Two edges are adjacent if they are incident with a common vertex. Two vertices u and v are adjacent if there is an edge e incident with both u and v. For every vertex $v \in V$, the set of all vertices adjacent to v is an *open neighborhood* of the vertex v denoted by $N(v)$ and the set $N[v] = N(v) \cup \{v\}$ is called the *closed neighborhood* of vertex v. The *degree* of a vertex v is the cardinality of its open neighborhood, denoted $d_G(v) = |N(v)|$. A vertex of degree one is called a *leaf* and its neighbor is called a *support vertex*. A support vertex with more than one leaf is called a *strong* support vertex and a support vertex with exactly one leaf is called a *weak* support vertex. The number of edges between u and v in a shortest path is the *distance* between vertices u and v. The longest distance between any pair of vertices is defined as the *diameter* of the graph G, and is denoted by $\mathrm{diam}(G)$. A path on n vertices is denoted by P_n. A vertex v in a tree T is adjacent to a path P_n through its vertex x, if a path containing x is one of the components of $(T - vx)$. A *star* of order $n \geq 2$, denoted by $K_{1,n-1}$, is a tree with at least $(n-1)$ leaves. A *double star* is a tree with exactly two support vertices and is denoted by $D_{r,s}$, where r and s are the number of leaves attached to each support vertices.

A subset S of V is a *dominating set* abbreviated DS, if every vertex in $V \setminus S$ is adjacent to some vertex in S. The *domination number*, $\gamma(G)$ of a graph G, is the minimum cardinality of a DS of G. By $\gamma(G)$-set, we mean a DS with minimum cardinality of a graph G.

A subset D of E is a *edge dominating set* abbreviated EDS, if every edge in $E \setminus D$ is adjacent to some edge in D. The *edge domination number*, $\gamma'(G)$ of a graph G, is the minimum cardinality of a EDS of G. By $\gamma'(G)$-set, we mean an EDS with minimum cardinality of graph G. For more properties on edge dominating set, refer the reader to [1, 7].

Characterizing trees with equal dominating parameters is available in the literature, see [6]. We characterize trees with domination number equal to twice edge domination number.

2. Main results

We begin this section with a theorem.

Theorem 1. *For any tree T, $\gamma'(T) \leq \gamma(T) \leq 2\gamma'(T)$.*

P r o o f. Let D be a $\gamma'(T)$-set. Let S be the set of vertices incident with the edges of D. The set S is a DS of tree T. Thus $\gamma(T) \leq |S| \leq 2|D| = 2\gamma'(T)$.

Let S be a $\gamma(T)$-set. For each vertex of S, select exactly one edge incident with it, and call such a set of edges as D. Then D is an EDS of tree T. We have $\gamma'(T) \leq |D| = |S| = \gamma(T)$.   □

For the purpose of characterizing trees with equal domination number and twice edge domination number, we introduce the family $\mathcal{A}$ of trees $T = T_k$ that can be obtained as follows.

Let $T_1 = P_4$. If $k \geq 2$, then T_{k+1} can be obtained recursively from T_k by one of the following operations:

- Operation $\mathcal{O}_1$: Attach a vertex to a support vertex of T_k.

- Operation $\mathcal{O}_2$: Attach a 4-path by joining its support vertex to a vertex of T_k adjacent to a 4-path through its support vertex.

- Operation $\mathcal{O}_3$: Attach a 4-path by joining its support vertex to a support vertex of T_k.

- Operation $\mathcal{O}_4$: Attach a double star $D_{r,s}$ with $r \cdot s \geq 2$ by joining a leaf adjacent to a strong support vertex to a vertex of T_k adjacent to a 2-path.

- Operation $\mathcal{O}_5$: Attach a double star $D_{r,s}$ with $r \cdot s \geq 2$ by joining a leaf adjacent to a strong support vertex to a support vertex of T_k.

The operations given above are illustrated in Figure 1. It is proved that $\gamma(T) = 2\gamma'(T)$ for every tree T of the family $\mathcal{A}$.

Lemma 1. *If $T \in \mathcal{A}$, then $\gamma(T) = 2\gamma'(T)$.*

P r o o f. To construct the tree T, we use the method of induction on the number k of operations. If $T = P_4$, then obviously $\gamma(T) = 2 = 2\gamma'(T)$. Let k be a positive integer. Assume that the result is true for every $T' = T_k$ of the family $\mathcal{A}$ constructed by $k - 1$ operations. Let $T = T_{k+1}$ be a tree of the family $\mathcal{A}$ constructed by k operations.

First assume that T is obtained from T' by operation $\mathcal{O}_1$. Let D' be a $\gamma'(T')$-set. It is easy to see that D' is an EDS of tree T. Thus $\gamma'(T) \leq \gamma'(T')$. Obviously, $\gamma(T') \leq \gamma(T)$. We now get
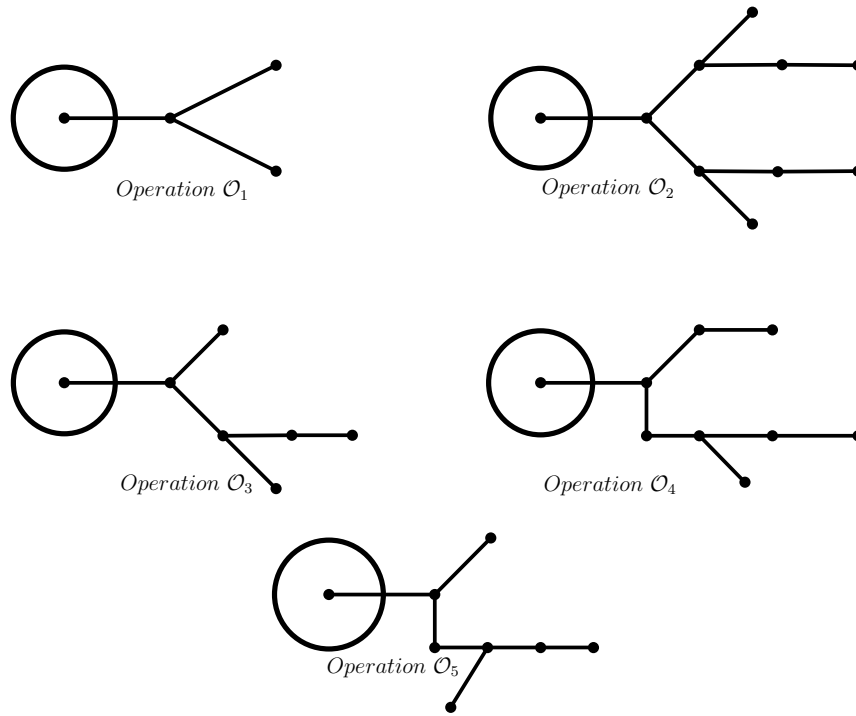
Figure 1. Operations $\mathcal{O}_1$ to $\mathcal{O}_5$

$2\gamma'(T) \leq 2\gamma'(T') = \gamma(T') \leq \gamma(T)$. On the other hand by Theorem 1, we have $\gamma(T) \leq 2\gamma'(T)$. This implies that $\gamma(T) = 2\gamma'(T)$.

Now assume that the tree T is obtained from the tree T' by the operation $\mathcal{O}_2$. Let x be the vertex to which a 4-path $pqrs$ is joined through q. Let q be adjacent to x. Let $abcd$ be a path different from path $pqrs$ with the vertex b adjacent to x. Let D' be a $\gamma'(T)$-set. To dominate the edges cd, bc, ab and bx, the edge $bc \in D'$. It is clear that $D' \cup \{qr\}$ is an EDS of T. Thus $\gamma'(T) \leq \gamma'(T') + 1$. Let S be a $\gamma(T)$-set. To dominate the vertices d, a, s and p, the vertices $c, b, r, q \in S$. It is obvious that $S \setminus \{q, r\}$ is a DS of tree T'. Thus $\gamma(T') \leq \gamma(T) - 2$. We obtain $2\gamma'(T) \leq 2\gamma'(T') + 2 = \gamma(T') + 2 \leq \gamma(T)$. We conclude that $2\gamma'(T) = \gamma(T)$.

Now assume that T is obtained from T' by the operation $\mathcal{O}_3$. Let x be the vertex to which the 4-path $pqrs$ is attached by joining q and x. Let y be a leaf adjacent to x. Let D' denote a $\gamma'(T')$-set. It is clear that $D' \cup \{q, r\}$ is an EDS of tree T. Thus $\gamma'(T) \leq \gamma'(T') + 1$. Let S be a $\gamma(T)$-set. To dominate the vertices s, p and y, the vertices $r, q, x \in S$. It is obvious that $S \setminus \{q, r\}$ is a dominating set of the tree T'. Thus $\gamma(T') \leq \gamma(T) - 2$. We now get $2\gamma'(T) \leq 2\gamma'(T') + 2 = \gamma(T') + 2 \leq \gamma(T)$. We conclude that $2\gamma'(T) = \gamma(T)$.

Now assume that T is obtained from T' by the operation $\mathcal{O}_4$. Let p and q be support vertices of a double star. Let r and s be two leaves adjacent to p, and t be the leaf adjacent to q. Denote by x the vertex to which the double star is attached. Let r be adjacent to x. Let x be adjacent to 2-path ab with a adjacent to x. Let D' be a $\gamma'(T')$-set. To dominate the edge ab, the edge $xa \in D'$. The set $D' \cup \{pq\}$ is an EDS of tree T. Thus $\gamma'(T) \leq \gamma'(T') + 1$. Let S be a $\gamma(T)$-set. To dominate the vertices t, s and b, the vertices $q, p, a \in S$. It is easy to observe $S \setminus \{p, q\}$ is a dominating set of the tree T'. Thus $\gamma(T') \leq \gamma(T) - 2$. We now get $2\gamma'(T) \leq 2\gamma'(T') + 2 = \gamma(T') + 2 \leq \gamma(T)$. We conclude that $\gamma(T) = 2\gamma'(T)$.

Assume that T is obtained from T' by the operation $\mathcal{O}_5$. Let p and q be support vertices of

the attached double star. Let r and s be two leaves adjacent to p, and t be a leaf adjacent to q. Denote by x the support vertex to which the double star is attached. Let r be adjacent to x. The leaf adjacent to x is denoted by y. Let D' represent a $\gamma'(T')$-set. The set $D' \cup \{pq\}$ is an EDS of the tree T. Thus $\gamma'(T) \leq \gamma'(T') + 1$. Let S be a $\gamma(T)$-set. The vertices t, s and y, are dominated by the vertices $p, q, x \in S$. The set $S \setminus \{p, q\}$ is a DS of the tree T'. Thus $\gamma(T') \leq \gamma(T) - 2$. We now get $2\gamma'(T) \leq 2\gamma'(T') + 2 = \gamma(T') + 2 \leq \gamma(T)$. We conclude that $\gamma(T) = 2\gamma'(T)$. $\qquad\square$

We now prove that if $2\gamma'(T) = \gamma(T)$, then the tree belongs to the family $\mathcal{A}$.

Lemma 2. *Let T be a tree. If $2\gamma'(T) = \gamma(T)$, then $T \in \mathcal{A}$.*

P r o o f. If $\mathrm{diam}\,(T) = 1$, then $T = P_2$. We have $\gamma(P_2) = 1 < 2 = 2\gamma'(P_2)$. If $\mathrm{diam}\,(T) = 2$, then T is a star. We have $\gamma(P_3) = 1 < 2 = 2\gamma'(P_3)$. If $\mathrm{diam}\,(T) = 3$, the tree T is a double star. If $T = P_4$, then $T \in \mathcal{A}$. If T is a double star other than P_4, then T is obtained from P_4 by required number of operations $\mathcal{O}_1$. Thus $T \in \mathcal{A}$. Let $\mathrm{diam}\,(T) \geq 4$. Thus the order n of the tree T is at least five. The method of induction on the order n is used to prove the result. Assume that the lemma is valid for every tree T' of order $n' < n$.

Assume that the support vertex of T, say x, is strong. Let p and q be leaves adjacent to x. Let $T' = T - p$. Let D be a $\gamma'(T)$-set. If $xp \in D$ then $(D \setminus \{xp\}) \cup \{xq\}$ is an EDS of T'. If $xp \notin D$ then obviously D is an EDS of T'. Thus $\gamma'(T') \leq \gamma'(T)$. Let S' be a $\gamma(T')$-set. Obviously S' is a DS of the tree T. Thus $\gamma(T) \leq \gamma(T')$. We now get $2\gamma'(T') \leq 2\gamma'(T) = \gamma(T) \leq \gamma(T')$. This implies that $\gamma(T') = 2\gamma'(T')$. We have $T' \in \mathcal{A}$ from the inductive hypothesis. The tree T is obtained from T' by operation $\mathcal{O}_1$. Thus $T \in \mathcal{A}$. Hereafter, it is assumed that every support vertex of T is weak.

Let r be a vertex of maximum eccentricity $\mathrm{diam}\,(T)$. We assume that r is the root of the tree T. The leaf at a maximum distance from r is denoted by t, t be the child of v, let v be the child of u in the rooted tree. If $\mathrm{diam}\,(T) \geq 4$, then let u be the child of w. If $\mathrm{diam}\,(T) \geq 5$, then let w be the child of d. If $\mathrm{diam}\,(T) \geq 6$, then let d be the child of e. The subtree induced by descendants of x and a vertex x in the rooted tree T is denoted by T_x.

Among the children of u assume that there is a support vertex, say x, other than v. Let y be the leaf adjacent to x. Let $T' = T - T_v$. Let D' be a $\gamma(T')$-set. The set $D' \cup \{v\}$ is a DS of T. Thus $\gamma(T) \leq \gamma(T') + 1$. Let S be a $\gamma'(T)$-set. To dominate the edges vt and xy, the edges $uv, ux \in S$. It is obvious that $S \setminus \{uv\}$ is EDS of the tree T'. Thus $\gamma'(T') \leq \gamma'(T) - 1$. We obtain

$$2\gamma'(T') \leq 2\gamma'(T) - 2 = \gamma(T) - 2 \leq \gamma(T') + 1 - 2 < \gamma(T').$$

By Theorem 1 this case is impossible.

Assume that some child of u, say x, is a leaf. By the choice of diametrical path, the vertex w is adjacent to isomorphic copy of T_u or adjacent to path P_3 or adjacent to path P_2 or a support vertex of T.

Case (i): Let w be adjacent to isomorphic copy of T_u, say $T_{u'}$. Let $T_{u'} = t'v'u'x'$. Let u' be adjacent to w. Let $T' = T - T_u$. Let D be a $\gamma'(T)$-set. To dominate the edges $vt, uv, ux, v't', u'v'$ and $u'x'$, the edges $uv, u'v' \in D$. It is obvious that $D \setminus \{uv\}$ is an EDS of T'. Thus $\gamma'(T') \leq \gamma'(T) - 1$. Let S' be a $\gamma(T')$-set. The set $S' \cup \{u, v\}$ is a DS of the tree T. Thus $\gamma(T) \leq \gamma(T') + 2$. We now obtain $2\gamma'(T') \leq 2\gamma'(T) - 2 = \gamma(T) - 2 \leq \gamma(T')$. This gives that $\gamma(T') = 2\gamma'(T')$. The tree $T' \in \mathcal{A}$ by the inductive hypothesis. The tree T can be constructed from T' by operation $\mathcal{O}_2$. Thus $T \in \mathcal{A}$.

Case (ii): Let w be adjacent to a 3-path abc. Let a be adjacent to w. Let $T' = T - T_a$. Let D be a $\gamma'(T)$-set. To dominate the edges vt, uv, ux, uw, wa, ab and bc, the edges $uv, ab \in D$. It is clear that $D \setminus \{ab\}$ is an EDS of the tree T'. Thus $\gamma'(T') \leq \gamma'(T) - 1$. Let S' be a $\gamma(T')$-set. The set $S' \cup \{b\}$ is obviously a DS of tree T. Thus $\gamma(T) \leq \gamma(T') + 1$. We now obtain

$$2\gamma'(T') \leq 2\gamma'(T) - 2 = \gamma(T) - 2 \leq \gamma(T') + 1 - 2 < \gamma(T').$$

By Theorem 1 this case is impossible.

Case (iii): Suppose w is adjacent to 2-path xy. Let w be adjacent to 2-path $x'y'$ different from 2-path xy. Let $T' = T - T_x$. Let D be a $\gamma'(T)$-set. To dominate the edges wx, xy, wx' and $x'y'$, the edges $wx, wx' \in D$. The set $D \setminus \{wx\}$ is an EDS of T'. Thus $\gamma'(T') \leq \gamma'(T) - 1$. Let S' be a $\gamma(T')$-set. The set $S' \cup \{x\}$ is clearly a DS of tree T. Thus $\gamma(T) \leq \gamma(T') + 1$. We now obtain $2\gamma'(T') \leq 2\gamma'(T) - 2 = \gamma(T) - 2 \leq \gamma(T') + 1 - 2 < \gamma(T')$. Suppose the vertex w is adjacent to exactly one 2-path xy. Let $T' = T - T_w$. Let D be a $\gamma'(T)$-set. The edges uv and wx are in D. The set $D \setminus \{uv, wx\}$ is an EDS of tree T'. Thus $\gamma'(T') \leq \gamma'(T) - 2$. Let S' be a $\gamma(T')$-set. It is obvious that $S' \cup \{u, v, x\}$ is a DS of tree T. Thus $\gamma(T) \leq \gamma(T') + 3$. We now get

$$2\gamma'(T') \leq 2\gamma'(T) - 4 = \gamma(T) - 4 \leq \gamma(T') + 3 - 4 < \gamma(T').$$

By Theorem 1 this case is impossible.

Case (iv): The vertex w is a support vertex. Let y be the leaf adjacent to w. Let $T' = T - T_u$. Let D be a $\gamma'(T')$-set. To dominate the edges vt, ux and wy, the edges $uv, e \in D$ where e is the edge incident with w other than wy. It is obvious that $D \setminus \{uv\}$ is an EDS of tree T'. Thus $\gamma'(T') \leq \gamma'(T) - 1$. Let S' be a $\gamma(T')$-set. The set $S' \cup \{u, v\}$ is obviously a DS of tree T. This gives $\gamma(T) \leq \gamma(T') + 2$. We now get $2\gamma'(T') \leq 2\gamma'(T) - 2 = \gamma(T) - 2 \leq \gamma(T')$. This implies that $2\gamma'(T') = \gamma'(T')$. The tree $T' \in \mathcal{A}$ by the inductive hypothesis. The tree T can be constructed from T' by operation $\mathcal{O}_3$. Thus $T \in \mathcal{A}$.

Case (v): Now assume $d_T(w) = 2$. By the choice of the diametrical path, the vertex d is adjacent to isomorphic copy of T_w or path P_4 or path P_3 or path P_2 or w is a support vertex or $d_T(d) = 2$.

Subcase (i): The vertex d is adjacent to isomorphic copy of T_w. Let D be a $\gamma'(T)$-set. To dominate the edges vt, uv, ux and uw the edge $uv \in D$. To dominate the edges in the isomorphic copy, the edges $u'v' \in D$. To dominate the edges incident with d, the edge $de \in D$. Let S be the set of vertices incident with edges in D. Clearly $|S| \leq 2|D|$. The set $S \setminus \{d\}$ is a DS of T. We have $\gamma(T) \leq 2|D| - 1 = 2\gamma'(T) - 1 < 2\gamma'(T)$.

Subcase (ii): The vertex d is adjacent to 4-path $a'b'c'd'$ with a' adjacent to d. Let D be a $\gamma'(T)$-set. As in subcase (i), the edge $uv \in D$. To dominate the edge da' and the edges in path $P_4 : a'b'c'd'$, the edges $de, b'c' \in D$. Let S be the set of vertices incident with edges in D. Clearly $|S| \leq 2|D|$. The set $S \setminus \{b'\}$ is a DS of T. We have $\gamma(T) \leq 2|D| - 1 = 2\gamma'(T) - 1 < 2\gamma'(T)$.

Subcase (iii): The vertex d is adjacent to 3-path $a'b'c'$ with a' adjacent to d. Let D be a $\gamma'(T)$-set. As in subcase (i), the edge $uv \in D$. To dominate the edge $b'c'$, the edge $a'b' \in D$. Let S be the set of vertices incident with edges in D. Clearly $|S| \leq 2|D|$. The set $S \setminus \{a'\}$ is a DS of T. We have $\gamma(T) \leq 2|D| - 1 = 2\gamma'(T) - 1 < 2\gamma'(T)$.

Subcase (iv): The vertex d is adjacent to a 2-path $a'b'$ with a' adjacent to d. Let $T' = T - T_w$. Let D be a $\gamma'(T)$-set. As in subcase (i), the edge $uv \in D$. To dominate the edges dw and $a'b'$, the edge $da' \in D$. It is easy to observe that $D \setminus \{uv\}$ is an EDS of T'. Thus $\gamma'(T') \leq \gamma'(T) - 1$. Let S' be a $\gamma(T')$-set. The set $S' \cup \{u, v\}$ is easily seen to be a DS of the tree T. Thus $\gamma(T) \leq \gamma(T') + 2$. We now obtain $2\gamma'(T') \leq 2\gamma'(T) - 2 = \gamma(T) - 2 \leq \gamma(T')$. This gives that $2\gamma'(T') = \gamma'(T')$. The tree $T' \in \mathcal{A}$ by the inductive hypothesis. The tree T can be constructed from T' by operation $\mathcal{O}_4$. Thus $T \in \mathcal{A}$.

Subcase (v): The vertex d is a support vertex. Let y be the leaf adjacent to d. Let $T' = T - T_w$. Arguing as in the previous subcase, we get $\gamma(T') = 2\gamma'(T')$. The tree $T' \in \mathcal{A}$ by the inductive hypothesis. The tree T can be constructed from T' by operation $\mathcal{O}_5$. Thus $T \in \mathcal{A}$.

Subcase (vi): Now assume the degree of the vertex d is two. Let D be a $\gamma'(T)$-set. To dominate the edges in T_w, the edge $uv \in D$. To dominate the edge dw, the edge $de \in D$. Let S

be the set of vertices incident with edges of D. Clearly $|S| \leq 2|D|$. The set $S \setminus \{d\}$ is a DS of the tree T. Thus $\gamma(T) \leq 2|D| - 1 = 2\gamma'(T) - 1 < 2\gamma'(T)$.

Now assume $d_T(u) = 2$. Let $T' = T - T_u$. Let D be a $\gamma'(T)$-set. To dominate the edge vt, the edge $uv \in D$. The set $D \setminus \{uv\}$ is verified to be an EDS of tree T'. Thus $\gamma'(T') \leq \gamma'(T) - 1$. Let S' be a $\gamma(T')$-set. The set $S' \cup \{v\}$ is obviously a DS of tree T. Thus $\gamma(T) \leq \gamma(T') + 1$. We now obtain $2\gamma'(T') \leq 2\gamma'(T) - 2 = \gamma(T) - 2 \leq \gamma(T') + 1 - 2 < \gamma(T')$. By Theorem 1 this case is impossible. $\qquad\qquad\square$

Characterization of trees with equal domination and twice the edge domination number is an immediate consequence of Lemma 1 and 2 and is stated as a theorem below.

Theorem 2. *Let T be a tree. Then $2\gamma'(T) = \gamma(T)$ if and only if $T \in \mathcal{A}$.*

3. Concluding remarks

In this paper we characterize trees with domination number equal to twice edge domination number and present some problems for further research, among them we note the following:

1. Characterize graphs with equal domination number and twice edge domination number.

2. Characterize trees with equal domination number and edge domination number.

3. Characterize graphs with equal domination number and edge domination number.

Acknowledgements

REFERENCES

1. Arumugam S., Velammal S. Edge domination in graphs. *Taiwanese J. Math.*, 1998. Vol. 2, No. 2. P. 173–179. URL: https://www.jstor.org/stable/43834391

2. Haynes T. W., Hedetniemi S. T., Slater P. J. *Fundamentals of Domination in Graphs.* New York: Marcel Dekker, 1998. 455 p.

3. Haynes T. W., Hedetniemi S. T., Slater P. J. *Domination in Graphs: Advanced Topics.* New York: Marcel Dekker, 1998. 497 p.

4. Krishnakumari B., Venkatakrishnan Y. B., Krzywkowski M. On trees with total domination number equal to edge-vertex domination number plus one. *Proc. Math. Sci.*, 2016. Vol. 126. P. 153–157. DOI: 10.1007/s12044-016-0267-6

5. Liu C. L. *Introduction to Combinatorial Mathematics.* New York: McGraw Hill, 1968. 393 p.

6. Dorfling M., Goddard W., Henning M. A., Mynhardt C. M. Construction of trees and graphs with equal dominating parameters. *Discrete Math.*, 2006. Vol. 306, No. 21. P. 2647–2654. DOI: 10.1016/j.disc.2006.04.031

7. Meddah N., Chellali M. Edges contained in all or in no minimum edge dominating set of a tree. *Discrete Math. Algorithms Appl.*, 2019. Vol. 11, No. 4. Art. no. 1950040. DOI: 10.1142/S179383091950040X

8. Venkatakrishnan Y. B., Krishnakumari B. An improved upper bound of edge-vertex domination number of a tree. *Inform. Process. Lett.*, 2018. Vol. 134. P. 14–17. DOI: 10.1016/j.ipl.2018.01.012