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ANALYSIS OF THE GROWTH RATE OF FEMININE MOSQUITO THROUGH DIFFERENCE EQUATIONS

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Abstract: The mosquito life cycle is developed mathematically with the concept of difference equation. The qualitative properties of the life-cycle are analyzed. The Lyapunov function is defined for difference equation to stabilize the system of mosquito life cycle. A novel technique is applied for deriving stability criterion, especially the back-stepping control technique is applied for discrete time system. The bifurcation analysis is also furnished for the model of mosquito life cycle. The new technique is applied in the mosquito life cycle model and its results are examined through MATLAB.

Keywords: Difference Equation, Mosquito, Bifurcation, Equilibrium, Strict Feedback.

1. Introduction

Research on mosquito epidemiology is imperative for the society. All over the world, all governments can pay more attention to mosquito epidemiological research. [1, 2, 4].

Many researchers developed a mathematical model of Plasmodium Life Cycle in Hepatocyte, mosquito midgut malaria transmission, HIV transmission, nitrogen cycle etc., in which the authors explore the complexity, bifurcation and analyze the stability of their model by the presence of an equilibrium point of the system [5, 6]. By constructing suitable conditions through the Lyapunov function, local and global stability analysis are discussed [7–9]. The difference equations have a long journey on the discrete time models of population dynamics [3]. These equations describe typically autonomous, discrete time dynamics and assume that there is only a temporary change in vital rates due to dependence on population density. An individual's important behaviour and activities can similarly change and fluctuate. Such kind of explicit dependencies on time can be modelled by using the difference equation. In the recent years, the difference equations have received more attention in the mathematical areas.

This paper is devotes a mathematical study of mosquito life cycle. The difference equation concept is utilized to construct the model. A novelty is involved in the derivation of stability conditions. Earlier researcher have not considered such type of Lyapunov function for difference equation. Section 2 describes the mathematical model for the mosquito life cycle under difference equation. Section 3 contains the discussion on equilibrium point position. Sections 4 includes the bifurcation analysis of the system of difference equation for the mosquito life cycle. In section 5 we investigate the stability analysis for the system with the conditions of Lypanouv stability, also related results are presented and finally, Section 6 describes the conclusion.

2. The mathematical model

The mathematical model for the Anopheles mosquito life cycle is described by the system of equations with the following assumptions.

- The total population of Anopheles mosquito life cycle consists of four forms, namely, adult, egg, larva and pupa.
- In every stage, the natural death rate μ is considered to be uniform.
- Let N denote the existing population, where ϕ is natural birth rate at adult stage.
- x_1 is the number of population existing at initial stage.
- x_2 is the number of eggs.
- x_3 is the population of larva.
- x_4 is the number of pupa.

The following Figure 1 shows the flow diagram of Anopheles mosquito life cycle.

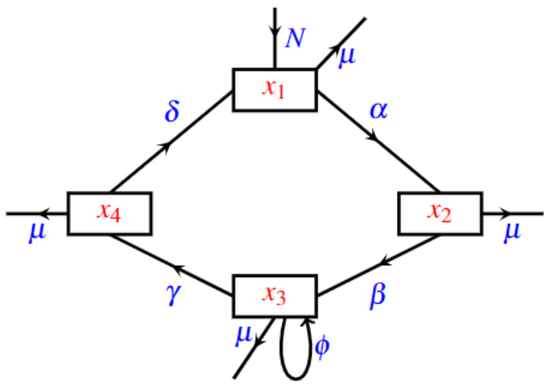

Figure 1. The flow diagram of Anopheles mosquito life cycle

The Anopheles mosquito life cycle is given by the following system of difference equation:

$$\begin{aligned} x_1(n+1) &= (N-\mu-\alpha)\ x_1(n)+\delta\ x_4(n), \\ x_2(n+1) &= \alpha\ x_1(n)-(\mu+\beta)\ x_2(n), \\ x_3(n+1) &= \beta\ x_2(n)-(\mu+\phi+\gamma)\ x_3(n), \\ x_4(n+1) &= \gamma\ x_3(n)-(\mu+\delta)\ x_4(n), \end{aligned} \tag{2.1}$$

where

- $x_1(n+1)$, $x_2(n+1)$, $x_3(n+1)$, $x_4(n+1)$ respectively are the difference equation at each stage,
- α, β, γ, δ are the respective rates of growth from one stage to another stage.

3. Analysis of equilibrium position

The equilibrium points are essential for analysing epidemiological dynamics which revolves around the equilibrium points. In epidemiology, the equilibrium point is a condition in which some identified or non-identified epidemiological form is balanced.

The epidemiological equilibrium points are unchanged from the epidemiological structure [10, 11]. They arise as a combination of corresponding epidemiological variables.

In mosquito epidemiology, adult, egg, lava and pupa are identified as key variables. The equilibrium points are obtained by means of relations

$$\begin{aligned} x_1^*(n) &= -\Big(\frac{\gamma}{N-\mu-\alpha}\Big)(x_4(n)),\\ x_2^*(n) &= -\Big(\frac{\alpha\gamma}{(N-\mu-\alpha)(\mu+\beta)}\Big)(x_4(n)),\\ x_3^*(n) &= -\Big(\frac{\gamma\alpha\delta}{(\mu+\beta)(N-\mu-\alpha)(\mu-\phi+\gamma)}\Big)(x_4(n)). \end{aligned} \tag{3.1}$$

If the pupa $x_4(n)$ state growth is equal to same arbitrary constant then the equilibrium points differ for following cases:

Case 1: If the arbitrary constant $\chi = 0$, then the four states of anopheles mosquito life cycle such as adult $x_1(n)$, eggs $x_2(n)$, larva $x_3(n)$ and pupa $x_4(n)$ are zero, which implies that a zero-equilibrium point.

Case 2: If the pupa growth rate is non-zero, also if

$$\chi > 0, \quad N-\mu-\alpha > 0, \quad \mu-\phi+\gamma > 0, \quad \mu+\beta > 0,$$

then $x_3 = -c_1$, $x_2 = -c_2$, $x_1 = -c_3$, and so $E = (-c_3, -c_2, -c_1, c_4)$ is an equilibrium solution.

Case 3: If

$$\chi < 0, \quad N-\mu-\alpha > 0, \quad \mu-\phi+\gamma > 0,$$

then $x_3 = c_1$, $x_2 = c_2$, $x_1 = c_3$, and so $E = (c_3, c_2, c_1, -c_4)$ is an equilibrium solution.

4. Bifurcation analysis

The purpose of bifurcation analysis is to study a dynamical system with respect to the trajectory represented by system, the occurrence of an equilibrium point and the stability properties of the equilibrium point, when changes occur in a certain parameter of the system of equations. The bifurcation analysis is carried out by linearizing the system of equations.

The Jacobian matrix is obtained as

$$\begin{bmatrix} (N-\mu-\alpha) & 0 & 0 & \delta \\ \alpha & -(\mu+\beta) & 0 & 0 \\ 0 & \beta & -(\mu-\phi+\gamma) & 0 \\ 0 & 0 & \gamma & -(\mu+\delta) \end{bmatrix}. \tag{4.1}$$

The characteristic equation of the above Jacobian matrix given by the equation (4.1) is obtained as

$$\Delta_1\lambda^4 + \Delta_2\lambda^3 + \Delta_3\lambda^2 + \Delta_4\lambda + \Delta_5 = 0,$$

where

$$\begin{aligned}
\Delta_1 &= 1,\\
\Delta_2 &= \alpha - N + b + \gamma + \delta + 4\mu - \phi,\\
\Delta_3 &= N\phi - N\gamma - Nd - 3N\mu - N\beta + \alpha\beta + \alpha\gamma + \alpha d + \beta\gamma + 3\alpha\mu + \beta d - \alpha\phi + 3\beta\mu\\
&\quad + \gamma\delta - \beta\phi + 3\gamma\mu + 3\delta\mu - \delta\phi - 3\mu\phi + 6\mu^2,\\
\Delta_4 &= 3\alpha\mu^2 - 3N\mu^2 + 3\beta\mu^2 + 3\gamma\mu^2 + 3\delta\mu^2 - 3\mu^2\phi + 4\mu^3 - N\beta\gamma - N\beta d - 2N\beta\mu\\
&\quad - N\gamma\delta + N\beta\phi - 2N\gamma\mu - 2N\delta\mu + N\delta\phi + 2N\mu\phi + \alpha\beta\gamma + \alpha\beta d + 2\alpha\beta\mu\\
&\quad + \alpha\gamma\delta - \alpha\beta\phi + 2\alpha\gamma\mu + \beta\gamma\delta + 2\alpha\delta\mu + 2\beta\gamma\mu - \alpha\delta\phi + 2\beta\delta\mu - 2\alpha\mu\phi\\
&\quad - \beta\delta\phi + 2\gamma\delta\mu - 2\beta\mu\phi - 2\delta\mu\phi,\\
\Delta_5 &= \alpha\mu^3 - N\mu^3 + \beta\mu^3 + \gamma\mu^3 + \delta\mu^3 - \mu^3\phi + \mu^4 - N\beta\mu^2 - N\gamma\mu^2 - N\delta\mu^2 + N\mu^2\phi\\
&\quad + \alpha\beta\mu^2 + \alpha\gamma\mu^2 + \alpha\delta\mu^2 + \beta\gamma\mu^2 + \beta\delta\mu^2 - \alpha\mu^2\phi + \gamma\delta\mu^2 - \beta\mu^2\phi - \delta\mu^2\phi - N\beta\gamma\delta\\
&\quad - N\beta\gamma\mu - N\beta\delta\mu + N\beta\delta\phi - N\gamma\delta\mu + N\beta\mu\phi + N\delta\mu\phi + \alpha\beta\gamma\mu + \alpha\beta\delta\mu - \alpha\beta\delta\phi\\
&\quad + \alpha\gamma\delta\mu - \alpha\beta\mu\phi + \beta\gamma\delta\mu - \alpha\delta\mu\phi - \beta\delta\mu\phi,
\end{aligned}$$

from the analysis with the different cases.

If any one of the parameter values is equal to zero or $N-\mu-\gamma < 0$ or $\mu+\beta < 0$ or $N-\mu-\phi-\gamma < 0$ or $\mu+\delta < 0$ then all the eigen values of the Jacobian matrix given in equation (4.1) are real. Hence for the linearised form of the system of equations there exists the hyperbolic equilibrium. Therefore the proposed mathematical model for the mosquito life cycle is satisfies the Lyapunov's conditions with respect to the robustness.

By introducing Holling type II parameter [15, 16] in larva stage ($x_3(n)$), the new dimension of the equation becomes,

$$x_3(n+1) = r\ x_3(n) - \left[0.2x_3^2(n) + \frac{0.375x_3(n)}{1+x_3^2(n)}\right],$$

where $r = -(\mu + \phi + \gamma)$ and the transmission rate from the state is

$$\beta x_2(n) = \left[0.2x_3^2(n) + \frac{0.375x_3(n)}{1+x_3^2(n)}\right].$$

The bifurcation exists at the larva state x_3 when the value of the parameter r varies between 2.5 and 4. Figure 2 shows the existence of bifurcation on the Anopheles mosquito life cycle at the larva state x_3.

5. Stability analysis of anopheles mosquito life cycle

In epidemiology the stability analysis of the system is possible to create a new example and explore new options. The stability analysis of anopheles mosquito life cycle is developing a balance of its cycle [12–14]. The following theorem gives the stability of the described model and the following relation establishes the condition for the anopheles mosquito life cycle.

Theorem 1. *The system of equation (2.1) for the anopheles mosquito life cycle is stabilized, if the following conditions exist for the system namely*

$$\begin{aligned}
(N-\mu-\alpha)x_1(n) &= x_1(n) - \delta x_4(n) - x_1^2(n+1),\\
(\mu+\beta)x_2(n) &= \alpha x_1(n) - x_2(n) + x_2^2(n+1),\\
(\mu-\phi+\gamma)x_3(n) &= \beta x_2(n) - x_3(n) + x_3^2(n+1),\\
(\mu+\gamma)x_4(n) &= \gamma x_3(n) - x_4(n) + x_4^2(n+1).
\end{aligned} \tag{5.1}$$

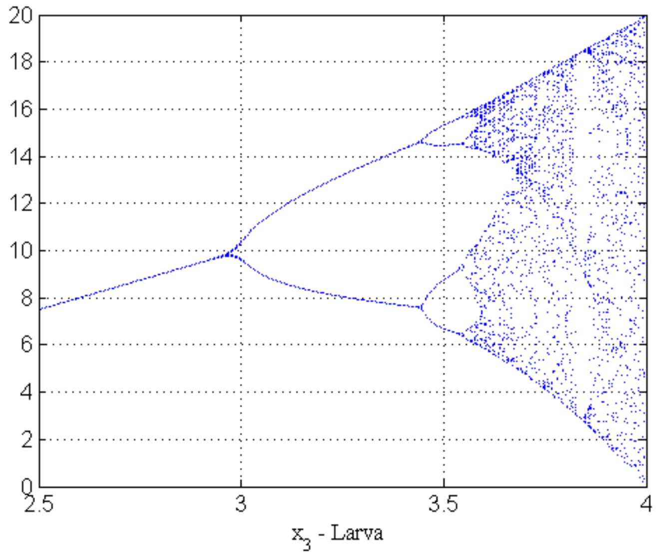

Figure 2. Existence of bifurcation in the Anopheles mosquito life cycle at the state x_3

P r o o f. Consider the Lyapunov function

$$V(x_n) = \sum_{i=1}^{4} (x_i(n)).$$

Take the difference equation (2.1), we obtain

$$\Delta V(x_n) = \sum_{i=1}^{4} \Delta\big(x_i(n)\big) \sum_{i=1}^{4} \big(x_i(n+1) - x_i(n)\big).$$

Substitutions of (5.1) in (2.1) leads to the relation

$$\Delta V = -x_i^2(n+1) \quad \text{for} \quad i = 1, 2, 3, 4.$$

Hence

$$\Delta V < 0,$$

which shows that V is a negative definite function. By Laselle's invariance principle, the model (2.1) is asymptotically stable. □

5.1. Stability analysis for Anopheles life cycle by using backward strict-feedback

The stability analysis helps to know how long the life can be accumulated and accelerated about the condition without any degradation. This study helps to determine the mean life of the mosquito. The strict-feedback control gives more accuracy to the system.

Theorem 2. *The system of equations* (2.1) *for the anopheles mosquito life cycle with the backward strict feedback mechanism under the concept of difference equation is globally asymptotically stable if*

$$\begin{aligned} u_1 &= (\mu + \delta + 1)x_4(n) - x_4^2(n), \\ u_2 &= -w_2^2(n), \\ u_3 &= -w_3^2(n), \\ u_4 &= -\delta x_4(n) - w_4^2. \end{aligned} \tag{5.2}$$

P r o o f. The backward strict feedback is applied to the system equation (2.1) to get the accuracy and so, consider the following difference equations

$$\begin{aligned} x_4(n+1) &= \gamma x_3(n) - (\mu+\delta)x_4(n) + u_1, \\ x_3(n+1) &= \beta x_2(n) - (\mu-\phi+\gamma)x_3(n) + u_2, \\ x_2(n+1) &= \alpha x_1(n) - (\mu+\beta)x_2(n) + u_3, \\ x_1(n+1) &= (N-\mu-\alpha)x_1(n) + \delta x_4(n) + u_4. \end{aligned}$$

Consider the stability of the pupa state

$$x_4(n+1) = \gamma x_3(n) - (\mu+\delta)x_4(n),$$

where $x_3(n)$ is regraded as a virtual controller.

Define the Lyapunov function

$$V_1(n) = x_4(n) \tag{5.3}$$

and the difference of the above equation (5.3) as follows

$$\Delta V_1(n) = \Delta x_4(n) = x_4(n+1) - x_4(n) = \gamma x_3(n) - (\mu+\delta)x_4(n) - x_4(n) + u_1. \tag{5.4}$$

Assume the virtual controller $x_3(n) = \kappa_1$ then we have

$$\Delta V_1(n) = \gamma\kappa_1 - (\mu+\delta)x_4(n) - x_4(n) + u_1.$$

By applying the controller,

$$u_1 = (\mu+\delta+1)x_4(n) - x_4^2(n)$$

and the virtual control $\kappa_1 = 0$ then the difference equation (5.4) becomes

$$\Delta V_1(n) = -x_4^2(n) < 0,$$

which is the negative definite function. Hence the pupa state x_4 is globally asymptotically stable.

Thus, the controller $\kappa_1(x_4(n))$ is an estimative when $x_4(n)$ is regarded as virtual controller.

The relation between x_3 and $k_1(x_4(n))$ is

$$w_2(n) = x_3(n) - \kappa_1.$$

Consider the $(x_4(n), w_2(n))$ subsystem (pupa and larva states)

$$\begin{aligned} x_4(n) &= -x_4(n) - x_4^2(n), \\ w_2(n+1) &= \beta\kappa_2 + w_2(n) + u_2. \end{aligned} \tag{5.5}$$

Let $x_2(n)$ be a virtual controller for the subsystem (5.5) and assume that the subsystem (5.5) is globally asymptotically stable when the state $x_2(n) = \kappa_2$.

Define the Lyapunov function

$$V_2(n) = x_4(n) + w_2(n).$$

The difference equation of $V_2(n)$ is

$$\Delta V_2(n) = \Delta x_4(n) + \Delta w_2(n) = x_4(n+1) - x_4(n) + w_2(n+1) - w_2(n). \tag{5.6}$$

Substituting the equation (5.5) in the difference equation (5.6), also taking $\kappa_2 = 0$ and $u_2 = -w_2^2(n)$, then the equation (5.6) leads to

$$\Delta V_2(n) = -x_4^2(n) - w_2^2(n).$$

Consequently V_2 is the negative definite function. Hence the system of equation (5.5) is globally asymptotically stable.

Thus, the function $w_2(n)$ is estimative, when the state $x_2(n)$ is consider as a virtual controller. Then the relation between $w_3(n)$ and $w_2(x_4(n), w_2(n))$ is

$$w_3(n) = x_2(n) - \kappa_2.$$

Consider the $(w_3(n), w_2(n), w_4(n))$ subsystem

$$\begin{aligned} w_3(n+1) &= \alpha x_1(n) + w_3(n) + u_3, \\ w_2(n+1) &= w_2(n) - w_2^2(n), \\ x_4(n+1) &= x_4(n) - x_4^2(n). \end{aligned} \tag{5.7}$$

Let $x_1(n)$ be a virtual controller in (5.7) and assume that the subsystem (5.7) is globally asymptotically stable, when $x_1(n) = \kappa_3$.

Let us define the Lyapunov function

$$V_3(n) = V_2(n) + w_3(n). \tag{5.8}$$

The differences from of the above equation (5.8) gives

$$\Delta V_3(n) = \Delta x_4(n) + \Delta w_2(n) + \Delta w_3(n). \tag{5.9}$$

Assume the controller $x_1(n) = \kappa_3$.

If $\kappa_3 = 0$, and $u_3 = -w_3^2(n)$, then the difference equation (5.9) leads to

$$\Delta V_3(n) = -x_4^2(n) w_2^2(n) - w_3^2(n) < 0,$$

which is the negative definite function. Hence the subsystem of equation (5.7) is globally asymptotically stable.

Thus, the function $w_4(n)$ is estimative when $x_1(n)$ is taking as virtual controller, then the relation between $x_1(n)$ and κ_3 is

$$w_4(n) = x_1(n) - \kappa_3.$$

Consider the $(w_4(n), w_2(n), w_3(n), w_4(n))$ subsystem

$$\begin{aligned} w_4(n+1) &= \gamma x_4(n) + w_4(n) + u_4, \\ w_3(n+1) &= w_3(n) - w_3^2(n), \\ w_2(n+1) &= w_2(n) - w_2^2(n), \\ x_1(n+1) &= x_1(n) - x_1^2(n). \end{aligned}$$

Let us assume the Lyapunov function is as follows

$$V_4(n) = V_3(n) + w_4(n). \tag{5.10}$$

The difference equation of $V_4(n)$ is

$$\Delta V_4(n) = \Delta x_4(n) + \Delta w_2(n) + \Delta w_3(n) + \Delta w_4(n). \tag{5.11}$$

Choose the controller as follows

$$u_4 = -\delta x_4(n) - w_4^2$$

substituting the controller u_4 in the equation (5.10), then the difference equation (5.11) becomes

$$\Delta V_4(n) = -x_4^2(n) - w_2^2(n) - w_3^2(n) - w_4^2(n) < 0,$$

which is negative definite function on $\mathbb{R}^4$. Thus by the concept of Lyapunov stability theory, the Anopheles mosquito life cycle (2.1) is globally asymptotically stable.

5.2. Numerical simulation

A numerical result is required in this section to validate the model's analytical result. MATLAB tool is utilised to confirm the theoretical results obtained in our model via backsteeping control technique analysis. Here the stability of the model is composed respect to two different initial conditions with the backstepping controllers is as follows in the system of equations (5.2).

The sensitive depend on initial condition is used to identify the stability and internal equilibrium that have a large influence on the each life cycle states.

To perform the sensitivity depend on initial conditions, the parameter values are considered as

$$\alpha = 0.341, \quad \beta = 0.567, \quad \gamma = 0.197, \quad \delta = 0.907.$$

The natural death rate $\mu = 0.4$ is considered to be uniform in all states and the total population N is considered as 10000000.

First, the initial conditions of the model is taken as

$$x_1(0) = 1.28, \quad x_2(0) = 8.76, \quad x_3(0) = 9.87, \quad x_4(0) = 8.23.$$

Figure 3 shows the stability on the internal equilibrium points. From Figure 3, the adult state x_1 is stable at 1.3869, the egg state x_2 is stable at 0.4063, the larva state x_3 is stable at 0.2019 and the pupa state x_4 is stable at 0.0305.

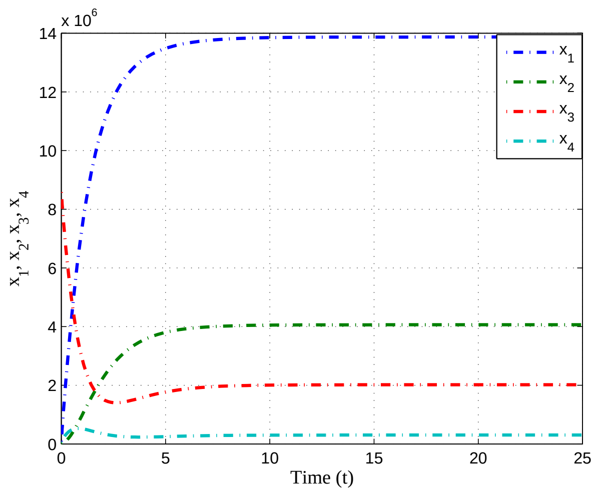

Figure 3. Stability at the internal equilibrium points

Second, the initial conditions of the model are taken as

$$x_1(0) = 86198, \quad x_2(0) = 27564, \quad x_3(0) = 8584367, \quad x_4(0) = 48975.$$

Figure 4 shows the stability on the internal equilibrium points. From the Figure 4, the adult state x_1 is stable at 1.3869, the egg state x_2 is stable at 0.4063, the larva state x_3 is stable at 0.2019 and the pupa state x_4 is stable at 0.0305.

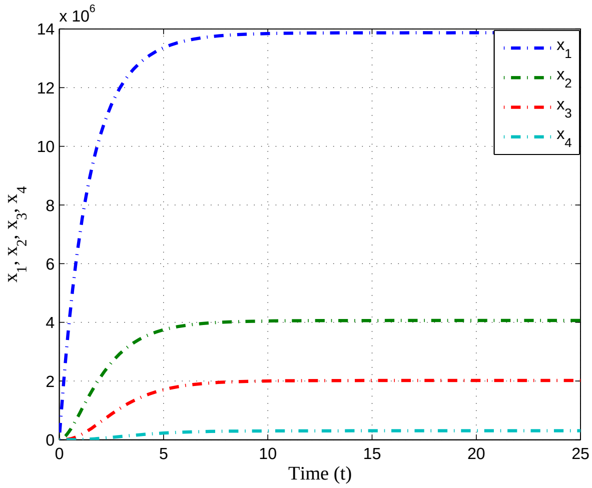

Figure 4. Stability on the internal equilibrium points

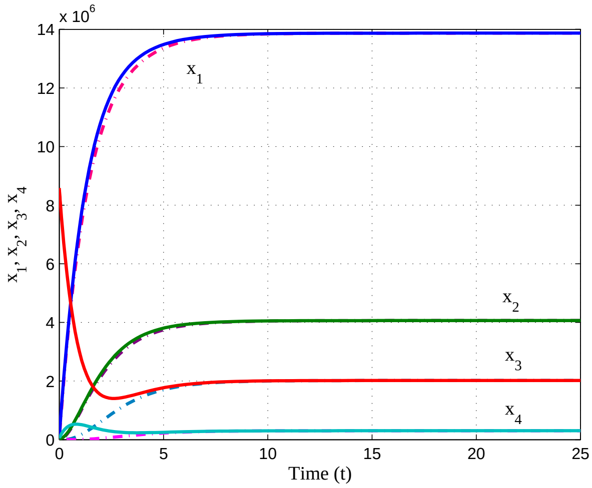

Figure 5. Sensitive dependance on initial conditions and internal equilibrium points

From the Figure 5, the Anopheles mosquito life cycle is stable at the internal equilibrium points, for this two different initial conditions were considered and the model is stable at the internal equilibrium points $x_1^*(n) = 1.3869,\ x_2^*(n) = 0.4063,\ x_3^*(n) = 0.2019,\ x_4^*(n) = 0.030$.

6. Conclusion

The Anopheles mosquito life cycle is modeled under the concept of difference equation. The stability of the model is estimated based on the Lyapunov conditions. The designing of the Lyapunov function is a new development in the difference equation concept. The strict feedback technique is also applied for a proposed mathematical model. Numerical results are furnished to supports the theory.

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