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GRAPHICAL PROPERTIES OF CLUSTERED GRAPHS

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Abstract: Clustering is a strategy for discovering homogeneous clusters in heterogeneous data sets based on comparable structures or properties. The number of nodes or links that must fail for a network to be divided into two or more sub-networks is known as connectivity. In addition to being a metric of network dependability, connectivity also serves as an indicator of performance. The Euler graph can represent almost any issue involving a discrete arrangement of objects. It can be analyzed using the recent field of mathematics called graph theory. This paper discusses the properties of clustered networks like connectivity and chromaticity. Further, the structure of the antipodal graph in the clustered network has been explored.

Keywords: Clustered graph, Euler graph, Antipodal graph.

1. Introduction

Clustering is a widely used technique to identify similar data or objects from the enormous one. It is used in genetic technology to group the genes with similar expression patterns [16]. Ustumbas [17] used tripartite graph clustering in a social network as a cluster of three types of data objects simultaneously and produced useful information for a recommended system.

A graph is commonly used to depict the architecture of an interconnection network. When constructing the topology of interconnection networks, there are various mutually exclusive needs [1]. Network dependability is an important consideration while planning the architecture of an interconnection network. The ability of a system to deliver services that can be legitimately trusted is known as dependability [2, 6]. A network's connectivity is defined as the number of nodes or links that must fail for the network to be partitioned into two or more distinct sub-networks. Network connection assesses a network's resilience and capacity to continue operating despite the presence of certain failing components. A higher node or link connection improves the network's resiliency to failure. Connectivity is not only a metric of network dependability but also a measure of performance. The number of links that must be crossed to reach a target node is reduced as connectivity improves. Because technical limits limit the number of connections per node to a limited value, designing a network with better connectivity and a consistent number of connections per node is critical.

It is implicitly assumed when using these measurements that any subset of the network components (channels or processors) might fail at the same moment. However, in certain networks, it is acceptable to assume that various subsets of network components do not break simultaneously. Classic connectivities may not be accurate gauges of dependability for these networks. Restricted connectivities may, of course, be used as a model to assess network dependability. Harary [9] developed the notion of conditional connection by demanding some characteristic for unconnected components of $G - F$, motivated by the inadequacies of existing connectivity measures. Esfahanian et. al [7] developed another similar study in which the authors provide extensions of edge connectivity by establishing specific constraints that detached components must meet. For applications where parallel algorithms might operate on sub-networks with a certain topology, requiring certain features for disconnected components is very critical. Restricting the defective sets to a certain class, on the other hand, was inspired by the fact that interconnection networks (which are typically represented as graphs) might include diverse components with varying levels of reliability [5, 8, 10, 13, 14, 18, 19].

In a computer network, which is known as digital telecommunications network, graphs are structured into clusters. Electronic links are employed within the same cluster where as optical links are employed between the cluster communications. These data links may be wire or optic cables or wireless media. Interconnection networks are strenuous to work with in abstract terms. This motivated many researchers to propose new improved network graphs arguing the benefits and performance evaluation in different contexts.

Yogalakshmi et. al [11] introduced the clustered graphs and found the degree-based topological indices for the same. Further, the interrelation of the indices was discussed. In [12], the distance-based topological indices of a clustered graph were computed. The clustered graph was derived from a complete tripartite graph $K_{r,s,t}$, $r \geq s \geq t$ with partite sets R, S, and T by converting each vertex as a complete graph with order equal to the degree of a corresponding vertex. The adjacency condition was preserved as in the tripartite graph.

This paper discusses the properties of clustered graphs such as connectivity, Euler property, antipodal graph, chromatic number, and chromatic index.

2. Preliminaries

Let $G = (V, E)$ be a connected simple graph with $|V| = p$ and $|E| = q$, where p and q are finite. Vertices u and v are adjacent if there is an edge $e \in E$ joining u and v, and the edge e is said to be incident with u and v. The number of edges incident with a vertex u is called the degree of the vertex and is denoted by $d(u)$. The maximum and minimum graph degrees are denoted by Δ and δ, respectively. The vertex connectivity of G denoted by $\kappa(G)$ and edge connectivity of G denoted by $\lambda(G)$ are the minimum number of vertices and edges, respectively, that need to be removed from a connected graph to make it disconnected. The distance between any two vertices u and v is the length of any shortest path connecting them. The distance to the vertex farthest away from a vertex u is its eccentricity $\epsilon(u)$. The lowest and greatest eccentricities are known as the radius $\mathfrak{R}(G)$ and diameter $\mathfrak{D}(G)$ of a graph, respectively. The central vertex is one with $\epsilon(u) = \mathfrak{R}(G)$. A vertex is diametrical or peripheral if $\epsilon(u) = \mathfrak{D}(G)$. A graph is self-centered if all its vertices are central. An Euler graph is a graph that contains an Eulerian circuit. A graph is Eulerian if and only if the degree of each vertex is even.

Definition 1 [4]. *A k-partite graph is one whose vertex set can be partitioned into k subsets, or parts so that no edge has both ends in the same part. A k-partite graph is complete if any two vertices in different parts are adjacent.*

Theorem 1 [4]. *A nontrivial connected graph G is Eulerian if and only if every vertex of G has an even degree.*

Definition 2 [3]. *The antipodal graph of a graph G denoted by $A(G)$ is the graph on the same vertices as G, two vertices being adjacent if the distance between is equal to the diameter of G. A graph is said to be antipodal if it is the antipodal graph $A(H)$ of some graph H.*

Definition 3 [15]. *A graph is called super-edge connected if every minimum edge cut consists of edges incident with a vertex of minimum degree.*

Definition 4 [7]. *The restricted edge connectivity $\lambda'(G)$ is the minimum cardinality of an edge cut S in a graph G with the property that $G - S$ contains no isolated vertices.*

Definition 5 [11]. *A cluster is an n-vertex graph with maximum adjacency between the vertices. A clustered set is the collection of all clusters that all have the same degree and there is no adjacency between the clusters.*

3. Properties of clustered graphs

Let $H = (V, E)$ be a graph, and let R, S, and T be clustered sets having r, s, and t clusters of maximum adjacency, respectively. Let vertices of the clustered sets R, S, and T of the clustered graph H be

$$\begin{aligned}
V(R) = \{&A_{11}, A_{12}, A_{13}, \dots, A_{1t}, A_{1(t+1)}, \dots, A_{1(t+s)}, A_{21}, A_{22}, \dots, A_{2t}, A_{2(t+1)}, \dots,\\
&A_{2(t+s)}, \dots, A_{r1}, A_{r2}, A_{r3}, \dots, A_{rt}, A_{r(t+1)}, \dots, A_{r(t+s)}\};\\
V(S) = \{&B_{11}, B_{12}, B_{13}, \dots, B_{1t}, B_{1(t+1)}, \dots, B_{1(t+r)}, B_{21}, B_{22}, \dots, B_{2t}, B_{2(t+1)}, \dots,\\
&B_{2(t+r)}, \dots, B_{s1}, B_{s2}, B_{s3}, \dots, B_{st}, B_{s(t+1)}, \dots, B_{s(t+r)}\};\\
V(T) = \{&C_{11}, C_{12}, C_{13}, \dots, C_{1s}, C_{1(s+1)}, \dots, C_{1(s+r)}, C_{21}, C_{22}, \dots, C_{2s}, C_{2(s+1)}, \dots,\\
&C_{2(s+r)}, \dots, C_{t1}, C_{t2}, C_{t3}, \dots, C_{ts}, C_{t(s+1)}, \dots, C_{t(s+r)}\}.
\end{aligned}$$

Then, $H = CL(r, s, t)$ is said to be a clustered graph if its vertex set V can be partitioned into three nonempty disjoint subsets $V(R)$, $V(S)$, and $V(T)$ of vertices of the clustered sets R, S, and T preserving the adjacency relation $(A_{i(t+j)}, B_{j(t+i)})$, (B_{jk}, C_{kj}), and $(C_{k(s+i)}, A_{ik})$ for $1 \le i \le r$, $1 \le j \le s$, and $1 \le k \le t$ as the complete tripartite graph $K_{r,s,t}$.

Now, the clustered graph $CL(r, s, t)$ is constructed as in Figure 1. The number of vertices n and the number edges m of the clustered graph are $2(rs + rt + st)$ and

$$\frac{1}{2}[r^2 s + r^2 t + s^2 r + s^2 t + t^2 r + t^2 s + 6rst],$$

respectively [11].

Example 1. Figure 2 shows the clustered graph $CL(3, 2, 2)$ as an example.

Theorem 2. *The connectivity number $\kappa(H)$ of a clustered graph H is $s + t$.*

P r o o f. By Whitney's inequality

$$\begin{aligned}
&\kappa(H) \le \lambda(H) \le \delta(H);\\
i.e.,\quad &\kappa(H) \le \lambda(H) \le s + t.
\end{aligned}$$

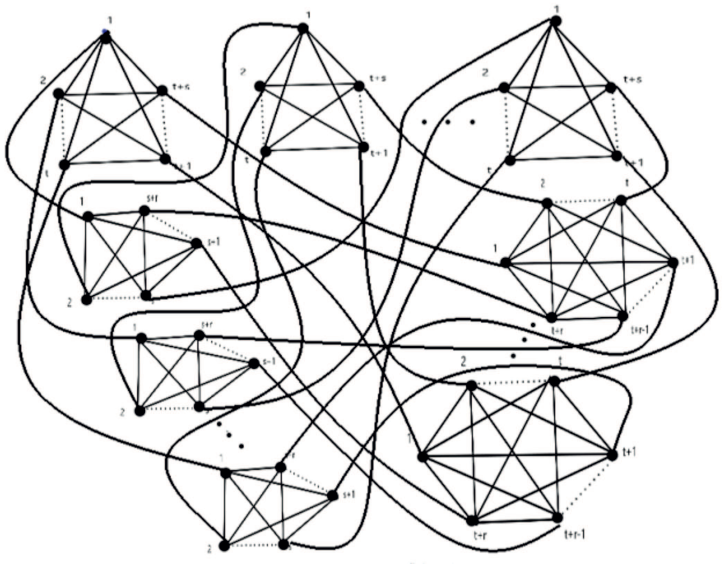

Figure 1. A clustered graph $CL(r, s, t)$.

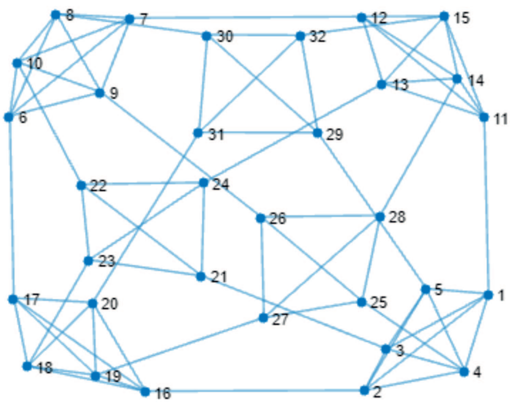

Figure 2. The clustered graph $CL(3, 2, 2)$.

Since the minimal degree is $s + t$, each cluster is adjacent to at least $s + t$ clusters, according to the adjacency criterion. As a result, detaching a cluster requires at least $s + t$ vertices. Therefore,

$$\kappa(H) = s + t.$$

□

Corollary 1. *The edge connectivity number $\lambda(H)$ of a clustered graph H is $s + t$.*

Theorem 3. *If $H(V, E)$ is a clustered graph and S is a subset of V, then*

$$2 \leq c(H - S) \leq |S|.$$

In other words, there are exactly two components in $(H - S)$ if $|S| = \delta(H)$.

P r o o f. The connectivity number of a clustered graph H is $s + t = \delta(H)$. Removal of $(s + t)$ vertices (or edges) disconnects only one cluster from the graph. Hence, by the adjacency condition of a clustered graph, the number of components is two. □

Corollary 2. *A clustered graph is a restricted edge- (λ')-connected graph.*

P r o o f. By Theorem 3, the number of components in $H - S$ is 2. Hence, it is λ'-connected. □

Corollary 3. *A clustered graph is super-edge connected.*

P r o o f. The clusters in the clustered set R contain vertices of minimum degree. By the structure of the clustered graph, the edges joining the vertices of each cluster in the clustered set R are edge cuts. Each edge in the edge cut is incident with a minimum degree vertex. Hence, the graph is super-edge connected. □

Theorem 4. *If each cluster in the clustered sets has minimum adjacency of all vertices of the clustered graph, then the minimum number of edges is* $[(rs + st + rt)(s + t)]$.

P r o o f. In a clustered graph H, we have

$$\kappa(H) = s + t \quad \text{and} \quad \kappa(H) \leq \delta(H).$$

Then the graph must have $(n/2)(s + t)$ edges, i.e., $[(rs + st + rt)(s + t)]$ edges. □

Theorem 5. *The distance between vertices in a clustered graph H is at most 4.*

P r o o f. For $1 \leq i \leq r$, $1 \leq l \leq s$, and $1 \leq k \leq t$, let A_i, B_l, and C_k be the clusters in the clustered sets R, S, and T, respectively:

$$\begin{aligned} R &= \{A_1, A_2, A_3, \ldots, A_i, \ldots, A_r\}, \\ S &= \{B_1, B_2, B_3, \ldots, B_l, \ldots, B_s\}, \\ T &= \{C_1, C_2, C_3, \ldots, C_k, \ldots, C_t\}. \end{aligned}$$

Vertices in each cluster are adjacent to each other. Any cluster in one of the clustered sets R, S, and T is adjacent to every cluster in the other clustered sets.

Consider a cluster A_i in the clustered set R. There are $(s + t)$ adjacent vertices in Ai, where s vertices are adjacent to s-clusters in S and t vertices are adjacent to t-clusters in T. Take a vertex v in A_i. Clearly, v is adjacent to a cluster B_l in S or C_k in T. Consequently, each vertex in B_l or C_k may be reached from v by a path of length at most 2. Furthermore, the path length grows by one from each vertex in A_i other than v. Similarly, the distance between a vertex in one clustered set and any vertex in another clustered set is at most three.

Now, consider vertices u in A_i and v in A_j with $i \neq j$. There is no adjacency between A_i and A_j for any $1 \leq i, j \leq r$. The path from u to v must pass through any cluster in S or T. The length of the $u - v$ path is 3 or 4.

Hence, the distance between vertices in the clustered graph is at most 4. □

Corollary 4. *In a clustered graph H, the radius is*

$$\mathfrak{R} = \begin{cases} 4, & r \geq s \geq t > 1, \\ 3, & \text{otherwise}, \end{cases}$$

and the diameter is

$$\mathfrak{D} = \begin{cases} 3, & r = s = t = 1, \\ 4, & \text{otherwise}. \end{cases}$$

Corollary 5. (i) *Clustered graph H is self-centered graph for either $r \geq s \geq t > 1$ or $r = s = t = 1$.*
(ii) *Complement of the clustered graph has diameter 2.*

Corollary 6. *In clustered graph antipodal vertices lies in the same clustered set.*

P r o o f. The diameter of clustered graph is 4. If v is a antipodal vertex of u then $d(u, v) = 4$. Let $u \in A_i$ in R then by Theorem 5, $v \in A_j$ in R for some j with $i \neq j$ such that $d(u, v) = 4$. Similarly it holds for the vertices in the clustered sets S and T.

Theorem 6. *The antipodal graph $A(H)$ of a clustered graph is disconnected, and the components are balanced partite graphs or isolated vertices.*

P r o o f. Case (i): $r = s = t = 1$. In this case, the clustered graph is isomorphic to C_6 and has a unique antipodal point for each vertex. Hence, the antipodal graph is disconnected.

Case (ii): $r \geq s \geq t > 1$. It is obvious that the clustered sets R, S, and T each have several clusters. According to Corollary 6, the antipodal vertex is in the same clustered set. Except for one vertex in each A_j, all vertices of the clusters A_j are the antipodal points of a vertex v in A_i, where $j \neq i$. Consequently, none of vertices in A_i is adjacent to the vertex v. It results in an r-partite graph, where r is the number of clusters in R. A similar argument demonstrates that the antipodal points of vertices in the clustered sets S and T form s-partite and t-partite graphs.

Case(iii): $r \geq s \geq t$ and $t = 1$. If $t = 1$, then the clustered set T contains only one cluster and its vertices have eccentricity 3. If $s = t = 1$, then both S and T contain one cluster each with vertices of eccentricity 3. Vertices in the clustered set R have eccentricity 4. The antipodal points does not exist for vertices with eccentricity 3. Since $V(A(H)) = V(H)$, the antipodal graph has isolated points. □

Theorem 7. *A clustered graph H is Eulerian if and only if r, s, and t are either all odd or all even.*

P r o o f. In a clustered graph, the possible degrees of vertices are $r + s$, $r + t$, and $s + t$. It is known that the sum of two numbers is even if both are either odd or even. According to the Eulerian criterion, the degree of all vertices is even if and only if r, s, and t are either all odd or all even. □

Corollary 7. *A clustered graph H is Eulerian if and only if its underlying complete tripartite graph is Eulerian.*

Theorem 8. *The complement $\bar{H}$ of a clustered graph H is not a Eulerian graph.*

P r o o f. The number of vertices in a clustered graph is $2(rs + rt + st)$, which is even. The degree of a vertex v in the complement graph is $\bar{d_v} = n - 1 - d_v$. It is clear that $\bar{H}$ is not Eulerian. □

Theorem 9. *If r, s, and t are neither all odd nor all even, then a spanning Eulerian subgraph exists.*

P r o o f. As the values of r, s, and t are neither all odd nor all even, two cases arise.

Case (i): there are *two odd and one even values.* Let r and s be odd and t be even. Let T be the clustered set containing clusters of degree $s + r$. Similarly, let the clustered sets S and R contain clusters of degree $t + r$ and $t + s$, respectively. Remove the edges adjacent with the clustered sets S

and R that lead to the degrees $t+r-1$ and $t+s-1$ of those adjacent vertices. There are t edges that arise from T to S and T to R, respectively. The vertices of the cluster B_1 in the clustered set S that are connected to T are $\{B_{11}, B_{12}, \dots, B_{1t}\}$, and we remove $t/2$ edges between the vertices alternatively. Continue this process for the remaining $s-1$ clusters in the clustered set S. Extend the same process to the clustered set R. Finally, the degrees of vertices are $s+r$, $t+r-1$, and $t+s-1$, which are even. Hence, the Eulerian spanning subgraph is derived.

Case (ii): there are *one odd and two even values.* Let r and s be even and t be odd. Let T, S, and R be clustered sets containing clusters of degree $s+r$, $t+r$, and $s+t$, respectively. The vertices $B_{i(t+j)}$, $i=1,2,\dots,s$, $j=1,2,\dots,r$, in the clustered set S are adjacent to the vertices $A_{j(t+i)}$ in the clustered set R. Remove the edges $A_{i(t+i)}B_{i(t+i)}$, $i=1,2,\dots,(s-1)$, and the edges $A_{i(t+s)}B_{s(t+i)}$, $i=s,(s+1),\dots,r$. The vertices B_{ij} and A_{kj}, $i=1,2,\dots,s$, $j=1,2,\dots,t$, $k=1,2,\dots,r$, are connected to the clustered set X. Remove the edges between these vertices (within the cluster) alternatively. The degrees of the vertices are $s+r$, $t+r-1$, and $t+s-1$, which are even. The proof is complete. ☐

Corollary 8. *A clustered graph H is a super-Eulerian graph for r, s, and t neither all odd nor all even.*

Theorem 10. *The chromatic number of a clustered graph is $s+r$.*

P r o o f. The maximum degree in a clustered graph is $s+r$. The bound for the chromatic number of a graph is $\Delta \le \chi \le \Delta+1$. Let the set of colors be $\{\alpha_1, \alpha_2, \dots, \alpha_i\}$, $s+r \le i \le s+r+1$.

Define a color function f as

$$\begin{array}{lll} f(C_{ij}) = \alpha_j, & 1 \le i \le t, \quad 1 \le j \le s+r, & \\ f(A_{ij}) = \alpha_j, & 1 \le i \le r, \quad 1 \le j \le t+s, & \\ f(B_{ij}) = \alpha_{(i+j)}, & (i+j) \mod (r+s), \quad 1 \le i \le s, \quad 1 \le j \le t+r. & \end{array}$$

Clearly, the range of f is $\{1,2,3,\dots,r+s\}$. Hence, the theorem is proved. ☐

Theorem 11. *The chromatic index of the clustered graph is $s+r$.*

P r o o f. By Vizing's theorem,

$$\chi'(H) \le 1 + \Delta(H).$$

Clearly, $\Delta(H) = r+s$; i.e.,

$$\chi'(H) \le 1 + r + s.$$

To prove the claim, it is enough to prove the existence of a function from the edge set of H to the color set $\mathcal{C}$, and $|\mathcal{C}| = r+s$. The edges can be categorized into two types by the structure of a clustered graph. One is within the clusters and another is between the clustered sets. Considering edges within the cluster, we obtain a complete graph. Moreover, the clusters in the clustered sets R, S, and T are of order $t+s$, $t+r$, and $s+r$, respectively. It is well known that

$$\chi'(K_n) = \begin{cases} \Delta, & n \text{ is even}, \\ \Delta+1, & n \text{ is odd}. \end{cases}$$

Case (i): $\Delta(H)$ is even. If $\Delta(H) = s+r$ is even, then the edges within the clustered set T are assigned $r+s-1$ colors. It is clear that the degrees of the clustered sets S and R are less than or equal to the degree of the clustered set T. Therefore, all the edges within the clusters of R, S, and T are assigned at most $r+s-1$ colors. The edges between the clustered sets are now assigned $(r+s)$th color. Hence, $\chi'(H) = r+s$.

Case (ii): $\Delta(H)$ is odd. Since $r+s$ is odd, the matching is not perfect and $r+s$ colors are needed. Let $\mathcal{C} = \{\alpha_1, \alpha_2, \ldots, \alpha_{s+r}\}$ be the color set, and let E be the edge set of the clustered graph. To assign colors between the clustered sets, define a function $f\colon E \to C$ as

$$\begin{array}{lll} f(C_{ij}B_{ji}) = \alpha_{p+i+j-2}, & (p+i+j-2) \bmod (r+s), & 1 \le i \le t, \quad 1 \le j \le s, \\ f(C_{i(s+j)}A_{ji}) = \alpha_{p+s+j+i-2}, & (p+s+j+i-2) \bmod (r+s), & 1 \le i \le t, \quad 1 \le j \le r, \\ f(B_{i(t+j)}A_{j(t+i)}) = \alpha_{p+t+i+j-2}, & (p+t+i+j-2) \bmod (r+s), & 1 \le i \le s, \quad 1 \le j \le r, \end{array}$$

where $p = (r+s+1)/2$.

In the clustered set T, edges adjacent to the clustered set R are assigned r colors, and s colors assign the edges adjacent to S with repetition of t clusters. There is a maximal matching of size $(s+r-1)/2$, and each cluster of T has $(r+s)(r+s-1)/2$ edges. Therefore, each cluster in the clustered set T is assigned $r+s$ colors. Since there is no adjacency between the clusters in the same clustered set, we assign the same $r+s$ colors to the clusters. Moreover, the clusters in the clustered set R and S are assigned at most $r+s$ colors. Thus, all the edges of H can be assigned at most $r+s$ colors. Hence, $\chi'(H) = r+s$. □

4. Conclusion

The concept of conditional connectivity is introduced in response to the shortcomings of the traditional connectivity measure by requiring some property for disconnected components. Similarly, edge connectivity was created by specifying certain measures of a network's robustness. Certain properties of disconnected components are required in applications where parallel algorithms can run on subnetworks with a given topology. Euler graph has reached the pinnacle of achievement in numerous circumstances arising in computer science, physical science, communication science, economics, and many other fields. It can be used to represent almost any issue involving discrete arrangements of objects where the focus is on the relationships between the objects rather than their internal properties. This paper discussed graph properties of clustered graphs like connectivity, Eulerian property, and chromaticity. Further properties of the clustered graphs will be incorporated in the future.

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