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PRICING POWERED α-POWER QUANTO OPTIONS WITH AND WITHOUT POISSON JUMPS

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Abstract: This paper deals with the problem of Black–Scholes pricing for the Quanto option pricing with power type powered and powered payoff underlying foreign currency is driven by Brownian motion and Poisson jumps, via risk-neutral probability measure. Our approach in this work is probabilistic, based on Feynman–Kac formula.

Keywords: Financial derivatives, Quanto option, Power payoff, Risk-neutral dynamics.

1. Introduction

This study focuses on the pricing of Quanto options with a powered-power payoff, where the underlying foreign currency is driven by a combination of Brownian motion and Poisson jumps, with the aim of avoiding arbitrage. Quanto options are derivatives that permit investors to acquire foreign assets without being exposed to the corresponding foreign exchange risk. These options are typically used when an investor wants to gain exposure to foreign assets without assuming foreign exchange risk [6]. For instance, if an investor wants to invest in a foreign market but does not want to take on the associated foreign exchange risk, they could utilize. Although, swap options remain a valuable strategic tool for the financial institutions by managing currency risks and exploring the opportunities from international markets. The main reason that traders buy and sell these assets is covering their risks in currency exposure, in addition to speculating that expected foreign currency appreciation will happen. So, this trading allows investors to take advantage of this real appreciation. Along with that, these financial instruments are multi-functional and cover more areas as portfolio diversification, tax optimization, the reduction of risk, etc. At times, a circumstance, where an investor tries to overcome currency problems and at the same time, manage their tax implications and portfolio diversification by using Quanto option which exists.

Conventionally, Quanto options have been solved using the Black–Scholes model as the underlying asset opinions under the guard condition of volatile constantly [1]. Although volatility imply method involves smiles and skews, however it is not a reason which leads to confusion. Addressing this, a series of local and volatility models are adopted, with a volatility, which is considered as a deterministic function of multiple factors such as the asset's price, underlying asset, current time, maturity, and option strike price. Local variability hypothesis of Quanto options addresses the accuracy of option price by overcoming the constraint of exogenously assumed volatility of options inherent in Black–Scholes model. In actual, Dupire [4] and Derman [3] were the leading researchers that developed and enhanced the permanent local volatility model as they identified a special diffusion process that is in line with the observed densities of the risk neutral probabilities which are derived from the implied volatility surfaces obtained from the European-style options in the market. The main advantage of local volatility models is their simplicity which is such that a randomness source is just one input, the price of underlying assets, thus giving the ability to easily calibrate. Here we have the power option that is a derivative in which the payoff is depended upon the underlying assets in the square root, cubic form, etc. Through this structure, the purchaser will be able to go for one side with respect to specific derivative or its volatility, or he will be left out depending on the trend observed in the Vanilla Options. Power options are commonly associated with the difference in the current price of the underlying instrument over fees that would exhibit intensity. Option in power call counterpart corresponds to cash flow of $\max(S_T^\alpha - K)$, while option in power put partners with $\max(K - S_T^\alpha)$, where $\alpha \geq 0, \alpha \in \mathbb{N}$. Taking Black–Scholes [1] leverage and diversification are the key discretionary using for only those investors who seek to acquire larger initial capital or premium, and, most possibly, this desire contributes to creation of their appeal.

Results are presented in this article are novel and have likely substantial value for the future comparisons of respective researches. Our work would encompass a variety of new findings at one point. The approach deals with gaps in the Black–Scholes risk-neutral valuation method, where the powered α-power Quanto call option prevails in the domestic currency which is fixed before and the use of the Feynman–Kac formula, both with and without Poisson jumps.

2. Price of Quanto option for a payoff at maturity

A foreign equity powered α-power Quanto call option, struck in a predetermined domestic currency, matures with a payoff given by

$$V_0 \left(\max\left(S_T^\alpha - K_f, 0\right)\right)^n = V_0 \left[\left(S_T^\alpha - K_f\right)^+\right]^n = V_0 \left[\left(S_T^\alpha - K_f\right)^n \mathbb{I}_{ST^\alpha > K_f}\right],$$

where V_0 represents a fixed exchange rate and K_f denotes the foreign currency strike price.

Assuming $n > 0$ is an integer, the payoff transforms into

$$V_0 \sum_{j=0}^{n} \binom{n}{j} \left(S_T^\alpha\right)^{n-j} \left(-K_f\right)^{1,j} \mathbb{I}_{\{S_T^\alpha > K_f\}}. \tag{2.1}$$

Theorem 1. *Let S_t represent the asset price in foreign currency X, and V_t denote the foreign exchange rate in foreign currency per unit of the domestic currency, both with constant volatilities σ_S and σ_V, respectively. We consider the risk-neutral dynamics (in domestic currency, cf. [5]) for a dividend-paying asset with rate q as follows*:

$$\begin{cases} dS_t = (r_f - q - \rho\sigma_S\sigma_V)S_t dt + \sigma_S S_t dB_t^{\mathbb{Q}^d}, \\ dV_t = (r_d - r_f)V_t dt + \sigma_V V_t dW_t^{\mathbb{Q}^d}, \end{cases} \tag{2.2}$$

where $B_t^{\mathbb{Q}^d}$ and $W_t^{\mathbb{Q}^d}$, $t \in [0, T]$, are $\mathbb{Q}^d$-standard Wiener processes. Then, for $\alpha > 0$, the price of a European power-α Quanto call option at time t in domestic currency with the payoff (2.1) is given by

$$C_q\left(t, S_t^\alpha\right) = V_0 e^{-r_d(T-t)} \sum_{j=0}^{n} \binom{n}{j} \left(-K_f\right)^{1,j} S_t^{\alpha(n-j)} e^{\alpha(n-j)\left\{r_f - q - \rho\sigma_S\sigma_V - (1-\alpha(n-j))\cdot\sigma_S^2/2\right\}\tau} N(d_{1,j}).$$

Here

$$d_{1,j} = \frac{\ln\left(S_t^\alpha / K_f\right) + \alpha\left(r_f - q - \rho\sigma_S\sigma_V - (1 + \alpha(n-j)/2)\,\sigma_S^2/2\right)\tau}{\alpha\sigma_S\sqrt{\tau}}.$$

P r o o f. Using the Feynman-Kac formula, as stated in Theorem 4.33 of the reference [2], the arbitrage price of a call option at time t, where t is less than or equal to the expiration date T, can be determined under the risk-neutral probability measure $\mathbb{Q}^d$,

$$C_q\left(t, S_t^\alpha\right) = V_0 e^{-r_d(T-t)} \sum_{j=0}^{n} \binom{n}{j} \left(-K_f\right)^{1,j} \mathbb{E}_{\mathbb{Q}^d}\left[\left(S_T^\alpha\right)^{n-j} \mathbb{I}_{\{S_T^\alpha > K_f\}} | \mathcal{F}_t\right]. \tag{2.3}$$

Hence, it remains to evaluate the conditional expectation in (2.3) for $0 \le j < n$. In order to compute that, we must compute the solution for the SDE (2.2). Applying Ito's lemma on process $(\ln S_t)$ for $t \ge 0$, hence

$$d\left(\ln S_t\right) = \left(r_f - q - \rho\sigma_S\sigma_V - \frac{\sigma_S^2}{2}\right) dt + \sigma_S dB_t^{\mathbb{Q}^d}.$$

Integrating both sides, we get,

$$\int_t^T d\left(\ln S_u\right) = \int_t^T \left(r_f - q - \rho\sigma_S\sigma_V - \frac{\sigma_S^2}{2}\right) du + \int_t^T \sigma_S dB_u^{\mathbb{Q}^d},$$
$$\ln\left(\frac{S_T}{S_t}\right) = \left(r_f - q - \rho\sigma_S\sigma_V - \frac{\sigma_S^2}{2}\right)(T-t) + \sigma_S\left(B_T^{\mathbb{Q}^d} - B_t^{\mathbb{Q}^d}\right),$$

i.e.

$$S_T = S_t e^{\left\{r_f - q - \rho\sigma_S\sigma_V - \sigma_S^2/2\right\}(T-t) + \sigma_S\left(B_T^{\mathbb{Q}^d} - B_t^{\mathbb{Q}^d}\right)}.$$

We then have

$$\left(S_T^\alpha\right)^{n-j} = S_t^{\alpha(n-j)} e^{\alpha(n-j)\left(r_f - q - \rho\sigma_S\sigma_V - \sigma_S^2/2\right)\tau - \alpha(n-j)\sigma_S\sqrt{\tau}Z}, \tag{2.4}$$

where

$$T - t = \tau \quad \text{and} \quad Z = -\frac{B_T^{\mathbb{Q}^d} - B_t^{\mathbb{Q}^d}}{\sqrt{\tau}} \sim \mathcal{N}(0,\, 1),$$

which is independent of $\mathcal{F}_t$, we find that $S_T^\alpha > K_f$ if and only if

$$Z < \frac{\ln\left(S_t^\alpha / K_f\right) + \alpha\left(r_f - q - \rho\sigma_S\sigma_V - \sigma_S^2/2\right)\tau}{\alpha\sigma_S\sqrt{\tau}} =: -d_{2,j}. \tag{2.5}$$

It follows from (2.4), (2.5) and from the independence of Z with $\mathcal{F}_t$ that

$$\mathbb{E}_{\mathbb{Q}^d}\left[\left(S_T^\alpha\right)^{n-j} \mathbb{I}_{\{S_T^\alpha > K_f\}} | \mathcal{F}_t\right] = S_t^{\alpha(n-j)} e^{\alpha(n-j)\left(r_f - q - \rho\sigma_S\sigma_V - \sigma_S^2/2\right)\tau}$$
$$\times \mathbb{E}_{\mathbb{Q}^d}\left[e^{-\alpha(n-j)\sigma_S\sqrt{\tau}Z} \mathbb{I}_{\{Z < d_{2,j}\}} | \mathcal{F}_t\right] = g(\tau, S_t^\alpha),$$

where $g(\tau, x)$ is given by

$$g(\tau, x) = x^{\alpha(n-j)} e^{\alpha(n-j)\left(r_f - q - \rho\sigma_S\sigma_V - \frac{\sigma_S^2}{2}\right)\tau} \mathbb{E}_{\mathbb{Q}^d}\left[e^{-\alpha(n-j)\sigma_S\sqrt{\tau}Z} \mathbb{I}_{\{Z < d_{2,j}\}} | \mathcal{F}_t\right].$$

Since $Z \sim \mathcal{N}(0,\, 1)$, we obtain

$$g(\tau, x) = x^{\alpha(n-j)} e^{\alpha(n-j)\left(r_f - q - \rho\sigma_S\sigma_V - \sigma_S^2/2\right)\tau} \int_{-\infty}^{d_{2,j}} \frac{1}{\sqrt{2\pi}} e^{-\alpha(n-j)\sigma_S\sqrt{\tau}z - z^2/2} dz$$
$$= x^{\alpha(n-j)} e^{\alpha(n-j)\left\{r_f - q - \rho\sigma_S\sigma_V - (1 - \alpha(n-j))\sigma_S^2/2\right\}\tau} \int_{-\infty}^{d_{2,j}} \frac{1}{\sqrt{2\pi}} e^{-(z + \alpha(n-j)\sigma_S\sqrt{\tau})^2/2} dz.$$

Applying the substituting $v = z + \alpha(n-j)\sigma_S\sqrt{\tau}$ and setting

$$\begin{aligned} d_{1,j} := d_{2,j} + \alpha(n-j)\sigma_S\sqrt{\tau} &= \frac{\ln\left(S_t^\alpha/K_f\right) + \alpha\left(r_f - q - \rho\sigma_S\sigma_V - \sigma_S^2/2\right)\tau}{\alpha\sigma_S\sqrt{\tau}} + \alpha(n-j)\sigma_S\sqrt{\tau} \\ &= \frac{\ln\left(S_t^\alpha/K_f\right) + \alpha\left(r_f - q - \rho\sigma_S\sigma_V - (1+\alpha(n-j)/2)\,\sigma_S^2/2\right)\tau}{\alpha\sigma_S\sqrt{\tau}}, \end{aligned} \tag{2.6}$$

we get

$$\begin{aligned} g(\tau, x) &= x^{\alpha(n-j)} e^{\alpha(n-j)\left\{r_f - q - \rho\sigma_S\sigma_V - (1-\alpha(n-j))\sigma_S^2/2\right\}\tau} \int_{-\infty}^{d_{1,j}} \frac{1}{\sqrt{2\pi}} e^{-v^2/2} dz \\ &= x^{\alpha(n-j)} e^{\alpha(n-j)\left\{r_f - q - \rho\sigma_S\sigma_V - (1-\alpha(n-j))\sigma_S^2/2\right\}\tau} N(d_{1,j}). \end{aligned} \tag{2.7}$$

From (2.6) and (2.7), (2.3) becomes

$$C_q\left(t, S_t^\alpha\right) = V_0 e^{-r_d(T-t)} \sum_{j=0}^{n} \binom{n}{j} \left(-K_f\right)^{1,j} S_t^{\alpha(n-j)} e^{\alpha(n-j)\left\{r_f - q - \rho\sigma_S\sigma_V - (1-\alpha(n-j))\sigma_S^2/2\right\}\tau} N(d_{1,j}),$$

where

$$d_1 = \frac{\ln\left(S_t^\alpha/K_f\right) + \alpha\left(r_f - q - \rho\sigma_S\sigma_V - (1+\alpha(n-j)/2)\,\sigma_S^2/2\right)\tau}{\alpha\sigma_S\sqrt{\tau}}.$$

Fig. 1 depicts the progression of the Quanto expense concerning maturity time T and the strike price K_f.

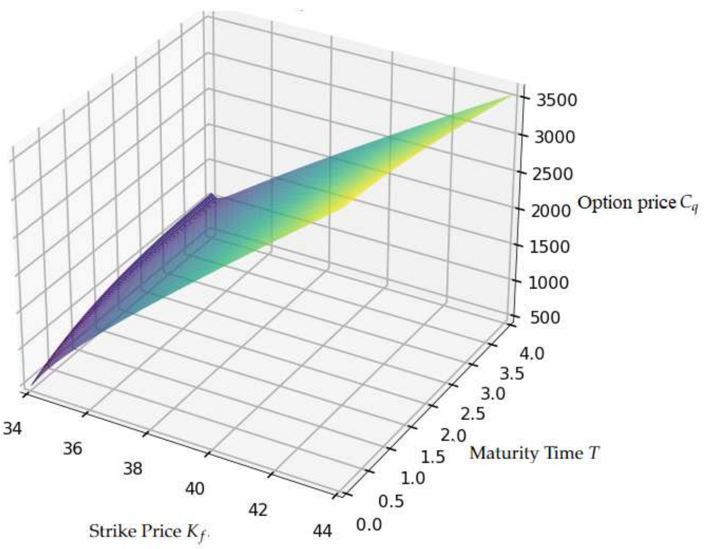

Figure 1. Powered power Quanto option value plotted against maturity time and strike price.

Now, our attention shifts to analyzing Quanto option premiums concerning the foreign currency strike price and maturity time. With $r_d = 0.5, V_0 = 30, \alpha = 5, r_f = 0.01, q = 0.1, \rho = 0.01, \sigma_s = 0.3$, and $\sigma_v = 0.2$, Fig. 1 illustrates the values of Quanto call option prices for $K_f \in [34, 44]$ and $T \in [0, 4]$. The plot reveals that while the evolution of Quanto option values isn't strictly monotonic, there's a discernible trend of increasing option prices with higher strike prices and longer maturities.

3. Pricing Quanto option with jumps

Theorem 2. *Suppose S_t represents the asset price in foreign currency X, where (N_t), $t \in \mathbb{R}+$, is a standard Poisson process with intensity $\lambda > 0$, independent of (B_t), $t \in \mathbb{R}+$, under a probability measure $\mathbb{Q}^d$. Let V_t denote the foreign exchange rate in foreign currency per unit of the domestic currency, both with constant volatilities σ_S and σ_V, respectively. We assume the following risk-neutral dynamics for a dividend-paying asset with rate q.*

$$\begin{cases} dS_t = (r_f - q - \rho\sigma_S\sigma_V)S_t dt + \sigma_S S_t dB_t^{\mathbb{Q}^d} + \eta S_{t-} dN_t, \\ dV_t = (r_d - r_f)V_t dt + \sigma_V V_t dW_t^{\mathbb{Q}^d}, \end{cases}$$

where $B_t^{\mathbb{Q}^d}$ and $W_t^{\mathbb{Q}^d}$, $t \in [0,T]$, are $\mathbb{Q}^d$ — standard Wiener processes. Then, for $\alpha > 0$, the price $C_q(t, S_t^\alpha)$ of a European power-α Quanto call option with jumps, at time t in domestic currency with the payoff (2.1), is given by,

$$C_q = V_0 e^{(\lambda - r_d)(T-t)} \sum_{j=0}^{n} \binom{n}{j} (-K_f)^{1,j} S_t^{\alpha(n-j)} e^{\alpha(n-j)\left\{r_f - q - \rho\sigma_S\sigma_V - (1-\alpha(n-j))\sigma_S^2/2\right\}\tau}$$
$$\times \sum_{n \geq 0} \frac{(\lambda(T-t))^n}{n!} N(d_{1,j}).$$

Here

$$d_{1,j} = \frac{\ln\left(S_t^\alpha(1+\eta)^n/K_f\right) + \alpha\left(r_f - q - \rho\sigma_S\sigma_V - (1+\alpha(n-j)/2)\,\sigma_S^2/2\right)\tau}{\alpha\sigma_S\sqrt{\tau}}.$$

P r o o f. As earlier, let us start by employing Feynman–Kac formula, as stated in [2, Theorem 4.33]. Under the risk-neutral probability measure $\mathbb{Q}^d$, the arbitrage price of the call option at time $t \leq T$ can be determined

$$C_q(t, S_t^\alpha) = V_0 e^{-r_d(T-t)} \sum_{j=0}^{n} \binom{n}{j} (-K_f)^{1,j}\, \mathbb{E}_{\mathbb{Q}^d}\left[(S_T^\alpha)^{n-j}\, \mathbb{I}_{\{S_T^\alpha > K_f\}} | \mathcal{F}_t\right], \tag{3.8}$$

where

$$S_T^\alpha = S_t^\alpha e^{\alpha\{r_f - q - \rho\sigma_S\sigma_V - \sigma_S^2/2\}(T-t) - \alpha\sigma_S(B_T^{\mathbb{Q}^d} - B_t^{\mathbb{Q}^d})}(1+\eta)^{N_T - N_t}.$$

We then have

$$(S_T^\alpha)^{n-j} = S_t^{\alpha(n-j)} e^{\alpha(n-j)\left(r_f - q - \rho\sigma_S\sigma_V - \sigma_S^2/2\right)\tau - \alpha(n-j)\sigma_S\sqrt{\tau}Z}(1+\eta)^{N_\tau}, \tag{3.9}$$

where

$$T - t = \tau, \quad Z = -\frac{B_T^{\mathbb{Q}^d} - B_t^{\mathbb{Q}^d}}{\sqrt{\tau}} \sim \mathcal{N}(0,1),$$

which is independent of $\mathcal{F}_t$, we find that $S_T^\alpha > K_f$ if and only if

$$Z < \frac{\ln\left(S_t^\alpha(1+\eta)^n/K_f\right) + \alpha\left(r_f - q - \rho\sigma_S\sigma_V - \sigma_S^2/2\right)\tau}{\alpha\sigma_S\sqrt{\tau}} =: -d_{2,j}. \tag{3.10}$$

It follows form (3.9), (3.10) and the independence of Z with $\mathcal{F}_t$ that

$$\mathbb{E}_{\mathbb{Q}^d}\left[(S_T^\alpha)^{n-j}\, \mathbb{I}_{\{S_T^\alpha > K_f\}} | \mathcal{F}_t\right] = S_t^{\alpha(n-j)} e^{\alpha(n-j)\left(r_f - q - \rho\sigma_S\sigma_V - \sigma_S^2/2\right)\tau}$$
$$\times \mathbb{E}_{\mathbb{Q}^d}\left[e^{-\alpha(n-j)\sigma_S\sqrt{\tau}Z}(1+\eta)^{N_\tau}\, \mathbb{I}_{\{Z < d_{2,j}\}} | \mathcal{F}_t\right] = g(\tau, S_t^\alpha),$$

where $g(\tau, x)$ is given by

$$g(\tau,x) = x^{\alpha(n-j)}e^{\alpha(n-j)\left(r_f-q-\rho\sigma_S\sigma_V-\sigma_S^2/2\right)\tau}\mathbb{E}_{\mathbb{Q}^d}\left[e^{-\alpha(n-j)\sigma_S\sqrt{\tau}Z}(1+\eta)^{N_\tau}\mathbb{I}_{\{Z<d_{2,j}\}}|\mathcal{F}_t\right]$$
$$= x^{\alpha(n-j)}e^{\alpha(n-j)\left(r_f-q-\rho\sigma_S\sigma_V-\sigma_S^2/2\right)\tau}\sum_{n\geq 0}\mathbb{P}(N_\tau=n)\mathbb{E}_{\mathbb{Q}^d}\left[e^{-\alpha(n-j)\sigma_S\sqrt{\tau}Z}(1+\eta)^{n}\mathbb{I}_{\{Z<d_{2,j}\}}|\mathcal{F}_t\right].$$

Since $Z \sim \mathcal{N}(0,\,1)$, we obtain

$$g(\tau,x) = x^{\alpha(n-j)}e^{\alpha(n-j)\left(r_f-q-\rho\sigma_S\sigma_V-\sigma_S^2/2\right)\tau}$$
$$\times\sum_{n\geq 0}\mathbb{P}(N_\tau=n)(1+\eta)^n\int_{-\infty}^{d_{2,j}}\frac{1}{\sqrt{2\pi}}e^{-\alpha(n-j)\sigma_S\sqrt{\tau}z-z^2/2}dz$$
$$= x^{\alpha(n-j)}e^{\alpha(n-j)\left\{r_f-q-\rho\sigma_S\sigma_V-(1-\alpha(n-j))\sigma_S^2/2\right\}\tau}e^{\lambda\tau}$$
$$\times\sum_{n\geq 0}\frac{(\lambda(T-t))^n}{n!}\int_{-\infty}^{d_{2,j}}\frac{1}{\sqrt{2\pi}}e^{-(z+\alpha(n-j)\sigma_S\sqrt{\tau})^2/2}dz.$$

Applying the substituting $v = z+\alpha(n-j)\sigma_S\sqrt{\tau}$ and setting

$$\begin{aligned} d_{1,j} &:= d_{2,j}+\alpha(n-j)\sigma_S\sqrt{\tau} \\ &= \frac{\ln\left(S_t^\alpha(1+\eta)^n/K_f\right)+\alpha\left(r_f-q-\rho\sigma_S\sigma_V-\sigma_S^2/2\right)\tau}{\alpha\sigma_S\sqrt{\tau}}+\alpha(n-j)\sigma_S\sqrt{\tau} \\ &= \frac{\ln\left(S_t^\alpha(1+\eta)^n/K_f\right)+\alpha\left(r_f-q-\rho\sigma_S\sigma_V-(1+\alpha(n-j)/2)\,\sigma_S^2/2\right)\tau}{\alpha\sigma_S\sqrt{\tau}}, \end{aligned} \tag{3.11}$$

we get

$$\begin{aligned} g(\tau,x) &= x^{\alpha(n-j)}e^{\alpha(n-j)\left\{r_f-q-\rho\sigma_S\sigma_V-(1-\alpha(n-j))\sigma_S^2/2\right\}\tau}e^{\lambda\tau}\sum_{n\geq 0}\frac{(\lambda(T-t))^n}{n!}\int_{-\infty}^{d_{1,j}}\frac{1}{\sqrt{2\pi}}e^{-v^2/2}dz \\ &= x^{\alpha(n-j)}e^{\lambda+\alpha(n-j)\left\{r_f-q-\rho\sigma_S\sigma_V-(1-\alpha(n-j))\sigma_S^2/2\right\}\tau}\sum_{n\geq 0}\frac{(\lambda(T-t))^n}{n!}N(d_{1,j}). \end{aligned} \tag{3.12}$$

From (3.11) and (3.12), (3.8) becomes

$$C_q = V_0e^{(\lambda-r_d)(T-t)}\sum_{j=0}^{n}\binom{n}{j}(-K_f)^{1,j}\,S_t^{\alpha(n-j)}e^{\alpha(n-j)\left\{r_f-q-\rho\sigma_S\sigma_V-(1-\alpha(n-j))\sigma_S^2/2\right\}\tau}$$
$$\times\sum_{n\geq 0}\frac{(\lambda(T-t))^n}{n!}N(d_{1,j}),$$

where

$$d_{1,j} = \frac{\ln\left(S_t^\alpha(1+\eta)^n/K_f\right)+\alpha\left(r_f-q-\rho\sigma_S\sigma_V-(1+\alpha(n-j)/2)\,\sigma_S^2/2\right)\tau}{\alpha\sigma_S\sqrt{\tau}}.$$

The diagram below illustrates the Quanto premium evolution with jumps concerning maturity time T and the strike price K_f.

Using the same dataset as before, with $r_d = 0.5, V_0 = 30, \alpha = 5, r_f = 0.01, q = 0.1,\ \rho = 0.01$, $\sigma_s = 0.3$, and $\sigma_v = 0.2$. Additionally, setting $\eta = 5, \lambda = 5, n = 6$ and $N = 5$, Fig. 2 depicts the progression of Quanto option prices with jumps. It's noticeable that the Quanto option value exhibits an upward trend concerning both variables, maturity time and strike price.

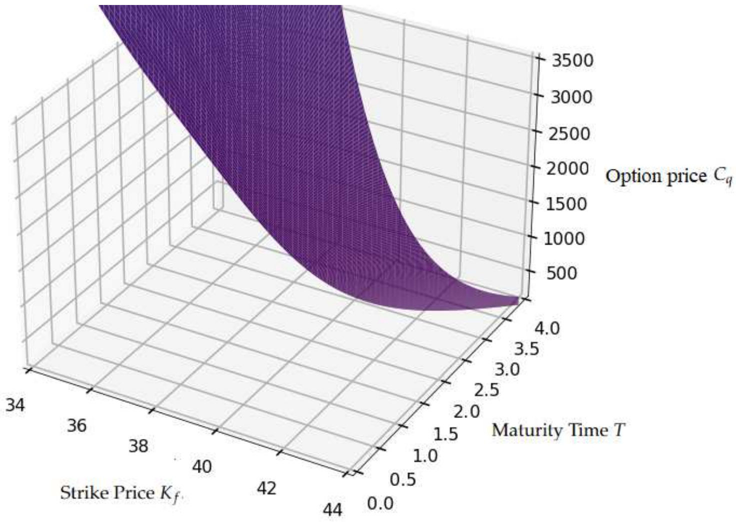

Figure 2. Powered power Quanto option call with jumps plotted against maturity time and strike price.

4. Conclusion

Quanto options are crucial tools for managing risk in the foreign exchange market. Determining their fair prices without arbitrage opportunities is essential. In this study, we have developed formulas to find the no-arbitrage prices for powered Quanto options. We considered scenarios where the underlying currencies follow Brownian motion and Brownian motion with jumps. We supported our theoretical framework with numerical simulations and results. We hope this research will inspire further exploration and interest in pricing exotic options.

REFERENCES

1. Black F., Scholes M. The pricing of options and corporate liabilities. *J. Political Economy*, 1973. Vol. 81, No. 3. P. 637–654. URL: http://www.jstor.org/stable/1831029
2. Capiński M., Kopp E., Traple J. *Stochastic Calculus for Finance.* Cambridge, UK: Cambridge University Press, 2012. 186 p.
3. Derman E., Kani I. Stochastic implied trees: arbitrage pricing with stochastic term and strike structure of volatility. *Int. J. Theor. Appl. Finance*, 1998. Vol. 1, No. 1. P. 61–110. DOI: 10.1142/s0219024998000059
4. Dupire B. Pricing with a smile. Risk, 1994. Vol. 7. P. 18–20.
5. Lee Y., Yoo H.-S., Lee J. Pricing formula for power Quanto options with each type of payoffs at maturity. *Global J. Pure Appl. Math.*, 2017. Vol. 13, No. 9. P. 6695–6702.
6. Teng L., Ehrhardt M., Günther M. The pricing of Quanto options under dynamic correlation. *J. Comput. Appl. Math.*, 2015. Vol. 275. P. 304–310. DOI: 10.1016/j.cam.2014.07.017