

ON THE OSCILLATION OF A THIRD ORDER NONLINEAR DIFFERENTIAL EQUATIONS WITH NEUTRAL TYPE

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Abstract: In this article, we investigate the oscillation behavior of the solutions of the third-order nonlinear differential equation with neutral type of the form

$$\left(a_1(t)(a_2(t)Z'(t))'\right)' + q(t)f(x(\sigma(t))) = 0, \quad t \geq t_0 > 0,$$

where $Z(t) := x(t) + p(t)x^\alpha(\tau(t))$. Some new oscillation results are presented that extend those results given in the literature.

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1. Introduction

Consider the third order non-linear neutral delay differential equation

$$\left(a_1(t)(a_2(t)Z'(t))'\right)' + q(t)f(x(\sigma(t))) = 0, \quad t \geq t_0 > 0, \quad (E)$$

where $Z(t) := x(t) + p(t)x^\alpha(\tau(t))$ and $0 < \alpha \leq 1$ is a ratio of odd positive integers. Throughout this paper, without further mention, let

(A₁) $a_i(t) \in C([t_0, +\infty))$, $a_i(t) > 0$ for $i = 1, 2$ and $p(t), q(t) \in C([t_0, +\infty))$, $q(t) > 0$;

(A₂) $\tau(t) \in C([t_0, +\infty))$, $\tau(t) \leq t$, $\sigma(t) \in C([t_0, +\infty))$, $\sigma(t) \leq t$;

(A₃) f is nondecreasing and $uf(u) \geq k > 0$ for $u \neq 0$ and $\lim_{t \rightarrow +\infty} \tau(t) = \lim_{t \rightarrow +\infty} \sigma(t) = \infty$.

By a solution of equation (E) we mean a nontrivial real valued function $x(t) \in C([T_x, \infty))$, $T_x \geq t_0$, which has the property $Z'(t) \in C^1([T_x, \infty))$, $a_2(t)Z'(t) \in C^1([T_x, \infty))$, $a_1(t)(a_2(t)Z'(t))' \in C^1([T_x, \infty))$ and satisfies (E) on $[T_x, \infty)$. We consider only those solutions $x(t)$ of (E) which satisfy $\sup\{|x(t)| : t \geq T\} > 0$ for all $T \geq T_x$. A solution of (E) is called oscillatory if it has arbitrarily large zeros on $[T_x, \infty)$ and otherwise, it is said to be non-oscillatory. Equation (E) is called almost oscillatory if all its solutions are oscillatory or convergent to zero asymptotically.

In the last years, a great deal of interest in oscillatory properties of neutral functional differential equations has been shown, we refer the reader to [1–8] and the references cited therein. A number

of authors including B. Baculiková and J. Džurina [10], T. Candan and Dahiya [11, 12], Graef et al. [7], and E. Thandapani and Li [8] have studied the oscillatory behavior of solutions of third order neutral differential equations in the form of equation (E) when $\alpha = 1$.

Recently, Lin and Tang [13] explored the oscillation of first-order neutral differential equation with a super-linear neutral term

$$[x(t) - p(x^\alpha(t - \tau))]' + q(t) \prod_{j=1}^m |x(t - \sigma_j)|^{\beta_j} \operatorname{sgn}[x(t - \sigma_j)] = 0,$$

where $\alpha > 1$. Ravi P. Agarwal et al. [14] concerned with oscillation of a certain class of second-order differential equations with a sub-linear neutral term

$$(a(t) [x(t) + p(t)x^\alpha(\tau(t))]' + q(t)x(\sigma(t))) = 0, \quad t \geq t_0 > 0,$$

where $0 < \alpha \leq 1$ is a ratio of odd positive integers and E. Thandapani et al. [9] established sufficient conditions for the oscillation of all solutions of a nonlinear differential equation

$$(a(t) [x(t) + p(t)x^\alpha(\tau(t))]' + q(t)x^\beta(\sigma(t))) = 0, \quad t \geq t_0 > 0,$$

where α and β are ratio of odd positive integers. The above observation shows that this paper extend the results in third order.

This article presents the further investigation of the oscillations of (E). The following two cases:

$$\int_{t_0}^{\infty} \frac{1}{a_1(t)} dt = \infty, \quad \int_{t_0}^{\infty} \frac{1}{a_2(t)} dt = \infty, \quad (1.1)$$

$$\int_{t_0}^{\infty} \frac{1}{a_1(t)} dt < \infty, \quad \int_{t_0}^{\infty} \frac{1}{a_2(t)} dt = \infty, \quad (1.2)$$

are studied.

The paper is organized as follows. In Section 2, we present sufficient conditions for the oscillation of all solutions of (E) and in Section 3, we provide some examples to illustrate the main results.

In the following, all functional inequalities considered in this paper are assumed to hold eventually, that is, they are satisfied for all t large enough. Without loss of generality, we can deal only with the positive solutions of (E).

2. Main result

In this section, we state and prove our main results for the equation (E). For convenience, we use the notations

$$p_*(t) = \left(1 - \frac{p(\sigma(t))}{M^{1-\alpha}}\right), \quad \Theta(t) = \frac{\int_{t_2}^{\sigma(t)} \left(\frac{1}{a_2(s)} \int_{t_1}^s \frac{du}{a_1(u)}\right) ds}{\int_{t_1}^t \frac{du}{a_1(u)}} \quad (2.1)$$

Theorem 1. *Let $0 \leq p(t) \leq p_1 \leq 1$. If (1.1) holds and it there exists a positive function $\phi \in C^1([t_0, \infty), \mathbb{R})$, such that for all sufficiently large $t_3 > t_2 > t_1 \geq t_0$ we have*

$$\limsup_{t \rightarrow \infty} \int_{t_3}^t \left(\phi(s)kq(s)p_*(s)\Theta(s) - \frac{a_1(s)(\phi'(s))^2}{4\phi(s)} \right) ds = \infty \quad (2.2)$$

and

$$\int_{t_0}^{\infty} \frac{1}{a_2(v)} \int_v^{\infty} \frac{1}{a_1(u)} \left[\int_u^{\infty} q(s) ds \right] du dv = \infty \quad (2.3)$$

holds for all constants $M > 0$, then (E) is almost oscillatory.

P r o o f. Suppose that $x(t)$ is a positive solution of (E). By condition (1.1), there exist two possible cases:

- (1) $Z(t) > 0$, $Z'(t) > 0$, $(a_2(t)Z'(t))' > 0$, $(a_1(t)(a_2(t)Z'(t)))' < 0$,
- (2) $Z(t) > 0$, $Z'(t) < 0$, $(a_2(t)Z'(t))' > 0$, $(a_1(t)(a_2(t)Z'(t)))' < 0$, for $t \geq t_1$, t_1 is large enough.

Assume $Z(t)$ satisfying property (1), then

$$\left(a_1(t)(a_2(t)Z'(t))' \right)' = -q(t)f(x(\sigma(t))) \leq -kq(t)x(\sigma(t)) < 0.$$

If there exists $t \geq t_1$ such that $Z(t) > 0$, $Z(\sigma(t)) > 0$, $Z'(t) > 0$, then $Z(t)$ is monotonically increasing, there exists a constant $M > 0$ such that $Z(t) \geq M$ and by the definition of Z we have

$$x(t) = Z(t) - p(t)x^\alpha(\sigma(t)) \geq Z(t) - p(t)Z^\alpha(\sigma(t)) \geq \left(1 - \frac{p(\sigma(t))}{M^{1-\alpha}} \right) Z(t) = p_*(t)Z(t), \quad (2.4)$$

where $p_*(t)$ is defined in (2.1). Let

$$\omega(t) = \phi(t) \frac{a_1(t)(a_2(t)Z'(t))'}{a_2(t)Z'(t)}, \quad (2.5)$$

$\omega(t) > 0$ for $t \geq t_1$. Differentiating (2.5), we obtain

$$\begin{aligned} \omega'(t) &= \phi'(t) \frac{a_1(t)(a_2(t)Z'(t))'}{a_2(t)Z'(t)} + \phi(t) \frac{(a_1(t)(a_2(t)Z'(t))')'}{a_2(t)Z'(t)} \\ &\quad - \phi(t) \frac{a_1(t)(a_1(t)(a_2(t)Z'(t))')(a_2(t)Z'(t))'}{(a_2(t)Z'(t))^2}. \end{aligned}$$

Since $(a_1(t)(a_2(t)Z'(t))')' < 0$, then $a_1(t)(a_2(t)Z'(t))'$ is decreasing, so

$$a_2(t)Z'(t) \geq \int_{t_1}^t \frac{a_1(s)(a_2(s)Z'(s))'}{a_1(s)} ds \geq a_1(t)(a_2(t)Z'(t))' \int_{t_1}^t \frac{ds}{a_1(s)},$$

which implies that

$$\left(\frac{a_2(t)Z'(t)}{\int_{t_1}^t ds/a_1(s)} \right)' \leq 0. \quad (2.6)$$

Thus,

$$Z(t) = Z(t_2) + \int_{t_2}^t \frac{a_2(s)Z'(s)}{\int_{t_1}^s \frac{du}{a_1(u)}} \frac{\int_{t_1}^s \frac{du}{a_1(u)}}{a_2(s)} ds \geq \frac{a_2(t)Z'(t)}{\int_{t_1}^t \frac{du}{a_1(u)}} \int_{t_2}^t \frac{\int_{t_1}^s \frac{du}{a_1(u)}}{a_2(s)} ds, \quad (2.7)$$

for $t \geq t_2 \geq t_1$.

It follows from (E), (2.4), and (2.5) that

$$\omega'(t) \leq \frac{\phi'(t)}{\phi(t)}\omega(t) - \frac{\omega^2(t)}{\phi(t)a_1(t)} - \phi(t)kq(t)p_*(t)\frac{Z(\sigma(t))}{a_2(t)Z'(t)},$$

that is,

$$\omega'(t) \leq \frac{\phi'(t)}{\phi(t)}\omega(t) - \frac{\omega^2(t)}{\phi(t)a_1(t)} - \phi(t)kq(t)p_*(t)\frac{Z(\sigma(t))}{a_2(\sigma(t))Z'(\sigma(t))} \frac{a_2(\sigma(t))Z'(\sigma(t))}{a_2(t)Z'(t)}.$$

From (2.6) and (2.7) follows

$$\begin{aligned} \omega'(t) &\leq \frac{\phi'(t)}{\phi(t)}\omega(t) - \frac{\omega^2(t)}{\phi(t)a_1(t)} - \phi(t)kq(t)p_*(t)\frac{\int_{t_2}^{\sigma(t)} \left(\frac{1}{a_2(s)} \int_{t_1}^s \frac{du}{a_1(u)} \right) ds}{\int_{t_1}^{\sigma(t)} \frac{du}{a_1(u)}} \frac{\int_{t_1}^{\sigma(t)} \frac{du}{a_1(u)}}{\int_{t_1}^t \frac{du}{a_1(u)}} \\ &= \frac{\phi'(t)}{\phi(t)}\omega(t) - \frac{\omega^2(t)}{\phi(t)a_1(t)} - \phi(t)kq(t)p_*(t)\frac{\int_{t_2}^{\sigma(t)} \left(\frac{1}{a_2(s)} \int_{t_1}^s \frac{du}{a_1(u)} \right) ds}{\int_{t_1}^t \frac{du}{a_1(u)}} \\ &\leq - \left[\frac{\omega(t)}{\sqrt{\phi(t)a_1(t)}} - \frac{1}{2} \sqrt{\frac{a_1(t)}{\phi(t)}} \phi'(t) \right]^2 - \phi(t)q(t)kp_*(t)\Theta(t) + \frac{a_1(t)(\phi'(t))^2}{4\phi(t)}, \end{aligned}$$

which implies

$$\omega'(t) \leq -\phi(t)q(t)kp_*(t)\Theta(t) + \frac{a_1(t)(\phi'(t))^2}{4\phi(t)}.$$

Integrating the last inequality from $t_3 (> t_2)$ to t we obtain

$$\int_{t_3}^t \left(\phi(s)q(s)kp_*(s)\Theta(s) - \frac{a_1(s)(\phi'(s))^2}{4\phi(s)} \right) ds \leq \omega(t_3).$$

Letting $t \rightarrow \infty$, it contradicts to (2.2).

Assume the case (2) holds. Using the similar proof of [10, Lemma 2], we can get $\lim_{t \rightarrow \infty} x(t) = 0$ due to condition (2.3). □

Theorem 2. *Let $0 \leq p(t) \leq p_1 \leq 1$. If (1.2) holds and there exists a positive function $\varphi \in C^1([t_0, \infty), \mathbb{R})$, such that for all sufficiently large $t_3 > t_2 > t_1 \geq t_0$, one has (2.2) and (2.3). If*

$$\limsup_{t \rightarrow \infty} \int_{t_2}^t \left(\delta(s)q(s)kp_*(s) \left(\int_{t_1}^{\sigma(s)} \frac{dv}{a_2(v)} \right) - \frac{1}{4\delta(s)a_1(s)} \right) ds = \infty, \tag{2.8}$$

where

$$\delta(t) := \int_t^\infty \frac{1}{a_1(s)} ds,$$

holds for all constants $M > 0$, then (E) is almost oscillatory.

P r o o f. Suppose that $x(t)$ is a positive solution of (E). By condition (1.2), there exist three possible cases:

- (1) $Z(t) > 0$, $Z'(t) > 0$, $(a_2(t)Z'(t))' > 0$, $(a_1(t)(a_2(t)Z'(t)))' < 0$,
- (2) $Z(t) > 0$, $Z'(t) < 0$, $(a_2(t)Z'(t))' > 0$, $(a_1(t)(a_2(t)Z'(t)))' < 0$, and
- (3) $Z(t) > 0$, $Z'(t) > 0$, $(a_2(t)Z'(t))' < 0$, $(a_1(t)(a_2(t)Z'(t)))' < 0$, for $t \geq t_1$, t_1 is large enough.

In cases (1) and (2) we can obtain the conclusion of Theorem 2 by applying the proof of Theorem 1. Assume that case (3) holds, $(a_1(t)(a_2(t)Z'(t)))' < 0$ and $a_1(t)(a_2(t)Z'(t))'$ is nonincreasing. Thus, we get

$$a_1(s)(a_2(s)Z'(s))' \leq a_1(t)(a_2(t)Z'(t))', \quad s \geq t \geq t_1.$$

Dividing the above inequality by $a_1(s)$ and integrating from t to l , we obtain

$$a_2(l)Z'(l) \leq a_2(t)Z'(t) + a_1(t)(a_2(t)Z'(t))' \int_t^l \frac{ds}{a_1(s)}.$$

Letting $l \rightarrow \infty$, we have

$$0 \leq a_2(t)Z'(t) + a_1(t)(a_2(t)Z'(t))' \int_t^\infty \frac{ds}{a_1(s)},$$

that is,

$$-\frac{a_1(t)(a_2(t)Z'(t))'}{a_2(t)Z'(t)} \int_t^\infty \frac{ds}{a_1(s)} \leq 1. \quad (2.9)$$

Now define φ as

$$\varphi(t) := \frac{a_1(t)(a_2(t)Z'(t))'}{a_2(t)Z'(t)}, \quad t \geq t_1. \quad (2.10)$$

Then $\varphi(t) < 0$ for $t \geq t_1$. Therefore, by (2.9) and (2.10), we obtain

$$-\delta(t)\varphi(t) \leq 1. \quad (2.11)$$

Differentiating (2.10) gives

$$\varphi'(t) = \frac{(a_1(t)(a_2(t)Z'(t))')'}{a_2(t)Z'(t)} - \frac{a_1(t)a_1'(t)(a_2(t)Z'(t))'(a_2(t)Z'(t))'}{(a_2(t)Z'(t))^2}.$$

Now $Z'(t) > 0$, so from (E) and (2.4) we have

$$\varphi'(t) \leq -q(t)kp_*(t) \frac{Z(\sigma(t))}{a_2(t)Z'(t)} - \frac{a_1(t)a_1'(t)(a_2(t)Z'(t))'(a_2(t)Z'(t))'}{(a_2(t)Z'(t))^2}. \quad (2.12)$$

In view of case (3), we see that

$$Z(t) \geq a_2(t) \int_{t_1}^t \frac{ds}{a_2(s)} Z'(t). \quad (2.13)$$

Thus, $\left(\frac{Z(t)}{\int_{t_1}^t ds/a_2(s)}\right)' \leq 0$ implies

$$\frac{Z(\sigma(t))}{Z(t)} \geq \frac{\int_{t_1}^{\sigma(t)} \frac{ds}{a_2(s)}}{\int_{t_1}^t \frac{ds}{a_2(s)}}. \tag{2.14}$$

From (2.10) and (2.12)–(2.14) we have

$$\varphi'(t) \leq -q(t)kp_*(t) \int_{t_1}^{\sigma(t)} \frac{ds}{a_2(s)} - \frac{\varphi^2(t)}{a_1(t)}.$$

Multiplying the above inequality by $\delta(t)$ and integrating it from $t_2 (> t_1)$ to t , we have

$$\begin{aligned} & \varphi(t)\delta(t) - \varphi(t_2)\delta(t_2) + \int_{t_2}^t \delta(s)kq(s)p_*(s) \left(\int_{t_1}^{\sigma(s)} \frac{dv}{a_2(v)}\right) ds \\ & + \int_{t_2}^t \frac{\varphi^2(s)\delta(s)}{a_1(s)} ds + \int_{t_2}^t \frac{\varphi(s)}{a_1(s)} ds \leq 0, \\ & \varphi(t)\delta(t) - \varphi(t_2)\delta(t_2) + \int_{t_2}^t \delta(s)kq(s)p_*(s) \left(\int_{t_1}^{\sigma(s)} \frac{dv}{a_2(v)}\right) ds - \int_{t_2}^t \frac{ds}{4\delta(s)a_1(s)} \\ & + \int_{t_2}^t \left[\sqrt{\frac{\delta(s)}{a_1(s)}}\varphi(s) + \frac{1}{2\sqrt{a_1(s)\delta(s)}} \right]^2 ds \leq 0, \end{aligned}$$

from which it follows that

$$\int_{t_2}^t \left(\delta(s)q(s)kp_*(s) \left(\int_{t_1}^{\sigma(s)} \frac{dv}{a_2(v)}\right) - \frac{1}{4\delta(s)a_1(s)} \right) ds \leq 1 + \varphi(t_2)\delta(t_2)$$

due to (2.11). Letting $t \rightarrow \infty$, we come to the contradiction (2.8). Then the result of the Theorem follows. □

3. Examples

In this section we will present some examples to illustrate the main results.

Example 1. Consider a third-order neutral differential equation

$$\left(t^{-1/2} \left(t^{1/2} \left[x(t) + \frac{1}{4}x^{3/5}(t-1) \right] \right)' \right)' + \frac{\lambda}{t^{1/2}}x(t-2) = 0, \quad t \geq 1, \tag{3.1}$$

where $\lambda > 0$ is a constant. Let $\alpha = 3/5$, $a_1(t) = t^{-1/2}$, $a_2(t) = t^{1/2}$, $p(t) = 1/4$, $q(t) = \frac{\lambda}{t^{1/2}}$, $\tau(t) = t - 1$, and $\sigma(t) = t - 2$. We obtain $p_*(t) = 1 - \frac{1/4}{M^{2/5}}$,

$$\int_{t_0}^{\infty} \frac{1}{a_2(v)} \int_v^{\infty} \frac{1}{a_1(u)} \left[\int_u^{\infty} \frac{\lambda}{s^{1/2}} ds \right] du dv = \infty.$$

and

$$\Theta(t) = \frac{\int_{t_2}^{\sigma(t)} \left(\frac{1}{a_2(s)} \int_{t_1}^s \frac{du}{a_1(u)} \right) ds}{\int_{t_1}^t \frac{du}{a_1(u)}} = \frac{(t-2)^3 + 6(t-2)^{1/2}t_1^{1/2} - 3c}{t^{3/2} - t_1^{3/2}},$$

where $c = t_2^3/3 + 2t_1^{3/2}t_2^{1/2}$. Pick $\phi(t) = 1$, then

$$\int_{t_3}^{\infty} q(s)p_*(s)\Theta(s)ds = \frac{\lambda}{3} \left(1 - \frac{1/4}{M^{2/5}} \right) \int_{t_3}^{\infty} \frac{(s-2)^3 + 6(s-2)^{1/2}t_1^{1/2} - 3c}{s^2 - s^{1/2}t_1^{3/2}} ds = \infty,$$

if $1/4 < M^{2/5}$. Hence, by Theorem 1, every solution of equation (3.1) is either oscillatory or converges to zero as $t \rightarrow \infty$ when $1/4 < M^{2/5}$.

Example 2. Consider a third-order neutral differential equation

$$\left(t^2 \left[x(t) + \frac{1}{2}x^{1/3}(t/8) \right]'' \right)' + \frac{1}{t} \left(1 + \frac{2}{27}t^{2/3} \right) x(t/2) = 0, \quad t \geq 1. \quad (3.2)$$

Let $\alpha = 1/3$, $a_1(t) = t^2$, $a_2(t) = 1$, $p(t) = 1/2$, $q(t) = 1/t(1 + 2/27t^{2/3})$, $\tau(t) = t/8$ and $\sigma(t) = t/2$. We obtain

$$p_*(t) = 1 - \frac{1/2}{M^{2/3}}, \quad \delta(t) = \int_t^{\infty} \frac{ds}{s^2} = \frac{1}{t}$$

and

$$\int_{t_1}^{t/2} \frac{ds}{a_2(s)} = \frac{1}{2}(t - 2t_1),$$

then

$$\begin{aligned} & \int_{t_2}^{\infty} \left(\delta(s)q(s)p_*(s) \left(\int_{t_1}^{\sigma(s)} \frac{dv}{a_2(v)} \right) - \frac{1}{4\delta(s)a_1(s)} \right) ds \\ &= \int_{t_2}^{\infty} \left(\left(1 - 0.5M^{-2/3} \right) \left[\frac{1}{2s} + \frac{1}{27}s^{-1/3} - \frac{t_1}{s^2} - \frac{2t_1}{27}s^{-4/3} \right] - \frac{1}{4s} \right) ds = \infty, \end{aligned}$$

if $0.5 < M^{2/3}$. Hence, by Theorem 2, every solution of equation (3.2) is either oscillatory or converges to zero as $t \rightarrow \infty$ when $0.5 < M^{2/3}$ and $x(t) = t^{-1}$ is such a solution of (3.2).

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