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THE IMPACT OF TOXICANTS IN THE MARINE THREE ECOLOGICAL FOOD-CHAIN ENVIRONMENT: A MATHEMATICAL APPROACH

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Abstract: To explore the impact of toxicants on a marine ecological food chain system consisting of three species, this work develops and analyzes a non-linear mathematical model. The model consists of five state variables: phytoplankton, zooplankton, fish, environmental toxicant, and organismal toxicant. We have incorporated the Monod-Haldane functional response as a predation function for each species. Using the Jacobian matrix, the stability analysis was conducted, and necessary constraints were obtained for the system's local and global stability. Hopf bifurcation analysis was performed for carrying capacity (K) and the rate of decrease in the growth rate of phytoplankton due to the presence of toxicants (r_1). Also, phase portraits are presented for different parameters of the model. In addition, numerical simulations are executed using MATLAB to prove theoretical findings and explore the impact of parameter variation on ecological species behavior.

Keywords: Environmental toxicant, Marine food chain, Stability, Hopf-bifurcation, Lyapunov function.

1. Introduction

It is well known that environmental contamination poses a significant threat to marine ecosystems. The main causes of it are industrial discharge and chemical spills. The rapid expansion of modern industry and agriculture significantly contributes to environmental pollution and habitat degradation. These pollutants contain harmful elements such as cadmium, zinc, copper, iron and mercury. As a result of the destruction of their natural ecosystems and increased exposure to dangerous pollutants, many species face serious risks to their survival, and many are on the verge of becoming extinct. Therefore, it is essential to study toxic substances in marine ecosystems from an environmental and conservational point of view.

In recent decades, mathematical models have become tremendously helpful in understanding and assessing the feeding relationships between species within ecosystems. In [2], Babu et al. explored the dynamic difficulties of a three-species food chain model. From the stability analysis, sufficient constraints for the survival and extinction of the population under toxicant stress have been revealed. Zhang et al. [22] considered an experimental marine food chain with three levels (microalgae → zooplankton → fish) to evaluate how feeding selectivity affects the transmission of methylmercury ($MeHg^{+}$) across the food chain system. In [11], Misra and Babu proposed and examined a three-species mathematical model in the presence of environmental and organismal toxicants. They found that Hopf bifurcation occurs at the predation rate of the intermediate predator. They also note that the system containing toxicants appears to be more stable than the toxicant-free system. Kalyan Das et al. [5] determine how the nanoparticle influences the interaction between phytoplankton and zooplankton. They observed that when zooplankton consumes phytoplankton, the growth of the zooplankton is slowed down by nanoparticles. Majeed and Kadhim [13] discussed the occurrence of local bifurcation and persistence under suitable food chain conditions, including a model of prey-first predator-second predator under the influence of toxins on all species. Talb et al. [20] considered a three-species aquatic food chain model in a polluted environment. It is noted that there are rich dynamics in the proposed food chain model, including periodic and chaotic. Kavita Yadav et al. [21] examined a marine tri-trophic food chain system that has distributed delay and environmental toxicants. They observed that distributed delay and environmental toxicants are crucial variables in the occurrence of Hopf bifurcation. Mandal et al. [14] created a mathematical model to study the control of the harmful effects of toxicants on the phytoplankton-zooplankton system by raising public awareness among people. They reveal that a moderate level of anthropogenic pollution might cause the phytoplankton-zooplankton system to become unstable. However, the contaminated system becomes stable due to public awareness. Smith and Weis [18] have observed that fish from polluted environments have much higher mortality rates than fish from unpolluted areas when they were exposed to a predator (blue crab Callinectes sapidus Rathbun).

Although several mathematical models may be used to explain the dynamics of interacting species, predator-prey theory is still based on the predator's functional response. Pal et al. [17] developed a simplified Monad Haldane (MH) functional response for toxin-producing phytoplankton and zooplankton populations and investigated how the toxication process of phytoplankton affects bloom creation and termination. Lui and Tan [9] where MH functional response is used for group defense theory. Several studies, based on theoretical and experimental data, have examined tri-trophic food chain systems, focusing on the impact of toxicants on the system's survival or extinction [1, 3, 4, 6–8, 10, 15, 16, 19]. So, these investigations encourage us to investigate the dynamics of the fish, phytoplankton, and zooplankton systems when toxicants are present.

In this paper, we formulated a mathematical model to study the impact of toxicants in a three-species marine food chain system considering Monad–Haldane functional responses. The existence of several equilibrium points has been examined. Then we established the local stability of the system using the Jacobian matrix. We also use the Lyapunov function and the Routh–Hurwitz criteria to assess the global stability and durability of the system.

2. Model formulation

Here, we consider an ecological model with three marine species. There are two ways through which toxicants can enter an organism. It can be absorbed by the population through resources (food chain) or directly from the environment. The model assumes that organismal toxicants have a negative impact on the growth rate of prey populations. In the absence of organismal toxicants, the prey's population growth follows logistic growth. In the model there are five state variables: $x(t)$ density of phytoplankton, $y(t)$ density of zooplankton, $z(t)$ density of fish, $c_e(t)$ concentration of environmental toxicants and $c_0(t)$ concentration of organism toxicant in the prey population. By considering these as state variables, we formulate a mathematical model to investigate the effects of toxicants on a three-species marine food chain system using the following system of non-linear ordinary differential equations

$$\frac{dx}{dt} = xr(c_0)\left(1 - \frac{x}{K}\right) - \frac{axy}{\alpha x^2 + m}, \tag{2.1}$$

$$\frac{dy}{dt} = \frac{bxy}{\alpha x^2 + m} - d_1 y - \frac{cyz}{\beta y^2 + h} - g_1 y^2, \tag{2.2}$$

$$\frac{dz}{dt} = \frac{dyz}{\beta y^2 + h} - d_2 z - g_2 z^2, \tag{2.3}$$

$$\frac{dc_e}{dt} = q_0 - a_1 c_e - a_2 x c_e + v x c_0, \tag{2.4}$$

$$\frac{dc_0}{dt} = a_2 x c_e - b_1 c_0 - v x c_0, \tag{2.5}$$

with $x(0) \geq 0,\ \ y(0) \geq 0,\ \ z(0) \geq 0,\ \ c_0 \geq 0,\ \ c_e(0) > 0$. Here, we assumed that the growth of phytoplankton is negatively affected by organismal toxicants, we consider

$$r(c_0) = r_0 - r_1 c_0,$$

where r_0 denotes the intrinsic growth rate of phytoplankton, r_1 is the constant that determines the rate of decrease in the growth rate of phytoplankton due to the presence of toxicants, and K is the environmental capacity.

The expression $axy/(\alpha x^2 + m)$ describes the predation of phytoplankton by zooplankton following Monad Haldane functional response, a is the predation rate, m is the saturation constant which is scaling the impact of the predator interference, food chain and food weighting factor, α denotes the inhibitory effect.

As the zooplankton population consumes the phytoplankton population, the growth is directly related to the rate at which phytoplankton is consumed, *i.e.*, response function for zooplankton is $bxy/(\alpha x^2 + m)$, where b is conversion coefficient, d_1 is the natural death rate of zooplankton and g_1 is the intraspecies competition coefficient among zooplankton population.

The term $cyz/(\beta y^2 + h)$ describes the predation of zooplankton by fish, c denotes the predation rate, h is the saturation constant which is scaling the impact of the predator interference, food chain and food weighting factor, and β denotes the inhibitory effect.

As zooplankton is consumed by the fish population, so the growth of fish is $dyz/(\beta y^2 + h)$, where d is the conversion coefficient of zooplankton to fish, d_2 is the natural death rate of fish population and g_2 is the intraspecies competition coefficient among fish population.

Let q_0 represents the external input of toxicant into the environment. The parameter v denotes the removal rate of a toxicant from the prey population (phytoplankton) due to its death. The parameter a_2 denotes the removal rate of a toxicant from the environment due to uptake by the phytoplankton (prey) populations. Furthermore, b_1 and a_1 denote the washout rates of organismal and environmental toxicant, respectively.

3. Boundedness of the Model

Determining the boundedness of solutions is essential to ensuring the system's biological feasibility. It guarantees that all population densities remain finite and non-negative for all time. Now we will determine the region of attraction, where our system is bounded.

Theorem 1. *Let the set*

$$\Omega = \Big\{(x, y, z, c_e, c_o) \in \mathbb{R}^5 :\ x(t) \leq K,\ x(t) + \frac{a}{b}y(t) + \frac{ac}{bd}z(t) \leq K_1,$$
$$c_e(t) + c_0(t) \leq K_2,\ c_e(t) \geq K_3,\ x(t) + c_e(t) \geq K_4\Big\},$$

then all solutions of the system are bounded in the region Ω*, where*

$$K_1 = \frac{(r_0+1)K}{\phi_1}, \quad K_2 = \frac{q_0}{\phi_2}, \quad K_3 = \frac{q_0}{a_1 + a_2 K}, \quad K_4 = \frac{(q_0 - aK_1)}{\phi_3},$$
$$\phi_1 = \min\{d, d_2, 1\}, \quad \phi_2 = \min\{a_1, b_1\}, \quad \phi_3 = \max\{r_1 K_2 - r_0, a_1 + a_2 K\}.$$

P r o o f. From (2.1), we get

$$\frac{dx}{dt} \leq xr_0\left(1-\frac{x}{K}\right).$$

By the usual comparison theorem, we get as $t \to \infty$,

$$x(t) \leq K.$$

Now, let us consider the following function:

$$F(t) = x(t) + \frac{a}{b}y(t) + \frac{ac}{bd}z(t)$$

by using (2.1), (2.2) and (2.3), we get

$$\frac{dF}{dt} + \phi_1 F \leq K(r_0+1),$$

where $\phi_1 = \min\{1,\ d,\ d_2\}$ then, by the usual comparison theorem, we get as $t \to \infty$

$$F(t) \leq \frac{K(r_0+1)}{\phi_1}, \quad F(t) = x(t) + \frac{a}{b}y(t) + \frac{ac}{bd}z(t) \leq K_1, \quad K_1 = \frac{K(r_0+1)}{\phi_1}.$$

Again, consider the following function:

$$G(t) = c_e(t) + c_0(t),$$

then by using (2.4), (2.5), we get

$$\frac{dG}{dt} + (a_1c_e + b_1c_0) \leq q_0,$$

then again using usual comparison theorem, we get as $t \to \infty$,

$$G(t) \leq \frac{q_0}{\phi_2},$$

where $\phi_2 = \min\{a_1,\ b_1\}$, and hence

$$c_e(t) + c_0(t) \leq K_2, \quad K_2 = \frac{q_0}{\phi_2}.$$

From (2.4) we get,

$$\frac{dc_e}{dt} + (a_1 + a_2K)c_e \geq q_0,$$

then, we get as $t \to \infty$,

$$c_e(t) \geq K_3, \quad K_3 = \frac{q_0}{a_1 + a_2K}.$$

Now let us consider the following function:

$$H(t) = x(t) + c_e(t),$$

by using (2.1) and (2.4) we get,

$$\frac{dH}{dt} + \phi_3 H \geq (q_0 - aK_1),$$

where

$$\phi_3 = \max\{r_1K_2 - r_0,\ a_1 + a_2K\},$$

then we get as $t \to \infty$,

$$H(t) \geq (q_0 - aK_1),$$

and hence,

$$x(t) + c_e(t) \geq K_4, \quad K_4 = \frac{(q_0 - aK_1)}{\phi_3}.$$

Hence, all the solutions of the system are bounded in the region Ω. □

4. Analysis of Model

4.1. Existence of equilibrium points

In steady-state solutions, where population densities do not change over time, the system's equilibrium points are found. These can be determined by solving the system of algebraic equations obtained by setting the right-hand sides of differential equations to zero. The set of four equilibrium points considered in this study includes all biologically feasible configurations of species survival and extinction under the influence of toxicants. Specifically, we examine: (*i*) the trivial equilibrium where no species survive, (*ii*) boundary equilibria representing partial survival of one or two species, and (*iii*) the interior equilibrium where all species coexist. Thus, the mathematical model has the following four positive equilibrium points, namely, $E_0(0,0,0,c_e,0)$, $\bar{E}_1(\bar{x},0,0,\bar{c_e},\bar{c_0})$, $\hat{E}_2(\hat{x},\hat{y},0,\hat{c_e},\hat{c_0})$, $E_3^\star(x^\star,y^\star,z^\star,c_e^\star,c_0^\star)$.

- *For the equilibrium point* $E_0(0,0,0,c_e,0)$:
  - from (2.4) we get $c_e = q_0/a_1$. When only an environmental toxicant is present, then the equilibrium point is $E_0(0,0,0,q_0/a_1,0)$.
- *In the absence of Zooplankton and Fish* $\bar{E}_1(\bar{x},0,0,\bar{c_e},\bar{c_0})$:
  - from (2.1) $\bar{x} = K$;
  - from (2.5) $\bar{c_0} = a_2 K\bar{c_e}/(b_1 + vK)$;
  - from (2.4)
  $$\bar{c_e} = \frac{q_0}{a_1 + a_2K - a_2vK^2/(b+vK)},$$
  $\bar{c_e} > 0$ if $(a_1 + a_2K)(b+vK) > a_2vK^2$.
- *In the absence of Fish* $\hat{E}_2(\hat{x},\hat{y},0,\hat{c_e},\hat{c_0})$:
  - from (2.2) we get
  $$\hat{y} = \frac{1}{g_1}\Big[\frac{b\hat{x}}{\alpha\hat{x}^2+m} - d_1\Big] \tag{4.1}$$
  $\hat{y} > 0$ if $b\hat{x} > (\alpha\hat{x}^2+m)d_1$;
  - from (2.4)
  $$\hat{c_e} = \frac{q_0(b_1+v\hat{x})}{(a_1+a_2\hat{x})(b_1+v\hat{x}) - va_2\hat{x}^2}$$
  $\hat{c_e} > 0$ provided $(a_1+a_2\hat{x})(b_1+v\hat{x}) > va_2\hat{x}^2$;
  - from (2.5)
  $$\hat{c_0} = \frac{a_2\hat{x}\hat{c_e}}{b_1+v\hat{x}}; \tag{4.2}$$
  - from (2.1) we get an algebraic equation in $\hat{x}$ variable,
  $$(r_0 - r_1\hat{c_0})(\alpha\hat{x}^2+m)\Big(1-\frac{\hat{x}}{K}\Big) - a\hat{y} = 0.$$

  A positive solution is obtained by solving the above equation for $\hat{x}$ and then the values of $\hat{c_0}$, $\hat{c_e}$, $\hat{y}$ can be computed from equations (4.1) to (4.2).
- *When all the species are present* (non-trivial equilibrium point) $E_3^\star(x^\star,y^\star,z^\star,c_e^\star,c_0^\star)$: the existence of the equilibrium point $E_3^\star$ has been established through the isocline method [12],

– from (2.1)

$$c_0^\star = \frac{K}{r_1(K-x)}\left[r_0\left(1-\frac{x}{K}\right) - \frac{ay}{\alpha x^2+m}\right] = m_1(x,y); \tag{4.3}$$

– from (2.4) and (2.5),

$$c_e^\star = \frac{1}{a_1}[q_0 - b_1 m_1(x,y)] = m_2(x,y);$$

– from (2.2),

$$z^\star = \frac{\beta y^2+h}{c}\left[\frac{bx}{\alpha x^2+m} - d_1 - g_1 y\right] = m_3(x,y). \tag{4.4}$$

Now, considering two functions (from (2.2) to (2.4)),

$$S_{11}(x,y) = q_0 - (a_1+a_2x)m_2(x,y) + vxm_1(x,y),$$

$$S_{12}(x,y)\frac{bdxy}{\alpha x^2+m} + vxm_1(x,y) + q_0 - d_1y(d+g_1y) - cz(d_2+g_2z) - (a_1+a_2x)m_2(x,y).$$

For the existence of $x^\star$ and $y^\star$, the two isoclines,

$$S_{11}(x,y) = 0, \tag{4.5}$$

$$S_{12}(x,y) = 0, \tag{4.6}$$

must intersect. We note that

$$S_{11}(0,0) = \frac{br_0}{r_1} > 0, \quad S_{12}(0,0) = \frac{br_0}{r_1} + hd_1d_2 - \frac{g_2h^2d_1^2}{c},$$

$$S_{12}(0,0) > 0 \quad \text{if} \quad \frac{br_0}{r_1} + hd_1d_2 > \frac{g_2h^2d_1^2}{c}.$$

Also considering, $S_{11}(x,0) = 0$ then x will be a positive root (say) ϕ_1, from the following value of x,

$$x = \frac{ba_1r_0}{a_2(br_0 - r_1q_0) - va_1r_0} > 0,$$

if $a_2(br_0 - r_1q_0) - va_1r_0 > 0$.
Now, consider $S_{11}(0,y) = 0$ then,

$$y = \frac{mr_0}{a} = \phi_2.$$

Now, let us consider $S_{12}(x,0) = 0$, then x will have one positive root (say) ϕ_3, from the following cubic equation of x,

$$\alpha Bx^3 + \alpha Ax^2 + (\alpha mB - bh)x + mA = 0,$$

if $\alpha mB < bh$ and $mA > 0$, where,

$$A = \frac{r_0b_1}{r_1} + d_1h > 0, \quad B = \left[\frac{r_0v}{r_1} - \frac{a_2}{a_1}\left(q_0 - \frac{b_1r_0}{r_1}\right)\right].$$

Now $S_{12}(0,y) = 0$, then y will have one positive root (say) ϕ_4, from the following equation of y,

$$A_1y^6 + A_2y^5 + A_3y^4 - A_4y^3 + A_5y^2 + A_6y - A_7 = 0,$$

$$A_1 = \frac{g_2\beta^2}{c}, \quad A_2 = \frac{2d_1g_1\beta^2g_2}{c}, \quad A_3 = \frac{2g_2\beta g_1^2h}{c} + \frac{g_2\beta^2d_1^2}{c},$$

$$A_4 = g_1d_2\beta - \frac{4g_2g_1d_1h\beta}{c}, \quad A_5 = \frac{2\beta hd_1^2g_2}{c} - \frac{g_1^2g_2h^2}{c} - d_1d_2\beta + g_1d_1,$$

$$A_6 = \frac{2g_1g_2d_1h^2}{c} - d_2hg_1 + dd_1 + \frac{ab_1}{r_1m}, \quad A_7 = \frac{b_1r_0}{r_1} + d_1d_2h - \frac{g_2d_1^2h^2}{c},$$

if $A_4 > 0$, $A_5 < 0$, A_6 and $A_7 > 0$. Thus, both the isoclines intersect each other in the region ω

$$\omega = \{(x, y) : 0 < x < \phi_3,\ 0 < y < \phi_2\},$$

in the following two cases (see Fig. 1):

$$(i): \quad \phi_3 > \phi_2, \quad \phi_1 > \phi_4,$$
$$(ii): \quad \phi_3 < \phi_2, \quad \phi_1 < \phi_4.$$

This point of intersection will give $x^\star$, $y^\star$. For the uniqueness of the $(x^\star, y^\star)$, we must have $dy/dx < 0$ for the curves in the region ω. For the curve (4.5),

$$\frac{dy}{dx} = \frac{(\alpha x^2 + m)}{aKF_2}\Big(F_1 r_1(K - x)(\alpha x^2 + m) - F_2 K\Big(-\frac{r_0(K - x)}{K} + \frac{2a\alpha xy}{\alpha x^2 + m} + A_8\Big)\Big) < 0, \quad (4.7)$$

where

$$F_1 = \frac{a_2}{a_1}(q_0 - b_1 m_1) - v m_1, \quad F_2 = \frac{a_1 + a_2 x}{a_1} b_1 + vx, \quad A_8 = r_0\Big(1 - \frac{x}{K}\Big) - \frac{ay}{\alpha x^2 + m}$$

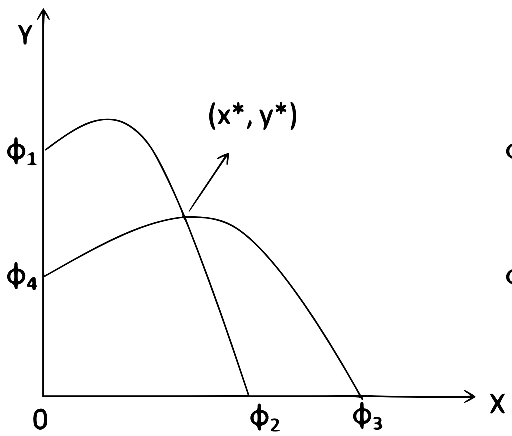
and for curve (4.6)

$$\frac{dy}{dx} = \frac{G_1 - G_2 - cm_3'(x, y)(d_2 + 2g_2 m_3) - bdy/(\alpha x^2 + m)}{d_1(d + 2gy) - bd/(\alpha x^2 + m)} < 0, \quad (4.8)$$

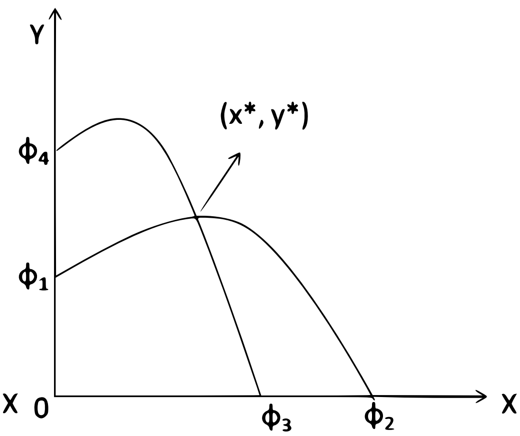
where

$$G_1 = m_1'(x, y)\Big[vx + \frac{b_1(a_1 + a_2 x)}{a_1}\Big], \quad G_2 = m_1(x, y)\Big[v + \frac{a_2 b_1}{a_1} - \frac{a_2 q_0}{a_1}\Big].$$

In case (i), the absolute value of dy/dx given by (4.7) is less than the absolute value of dy/dx given by (4.8). For the case (ii), the condition is the opposite. Knowing the value of $x^\star$, $y^\star$; $z^\star$, $c_e^\star$ and $c_0^\star$ can be computed from the (4.3) to (4.4).



Case (i): $\phi_3 > \phi_2$, $\phi_1 > \phi_4$.



Case (ii): $\phi_3 < \phi_2$, $\phi_1 < \phi_4$.

Figure 1. Existence of equilibrium point $E_3^\star$ of the Model.

4.2. Local stability of the Model

Local stability analysis investigates the behavior of solutions in proximity to equilibrium points through the examination of the Jacobian matrix. To validate the local stability of the equilibrium, the eigenvalues of the Jacobian matrix are computed at each equilibrium point. If all eigenvalues have a negative real part, the equilibrium point is locally asymptotically stable.

The Jacobian matrix associated with the Model is

$$J = \begin{bmatrix} d_{11} & -d_{12} & 0 & -d_{13} & 0 \\ d_{21} & -d_{22} & -d_{23} & 0 & 0 \\ 0 & d_{32} & d_{33} & 0 & 0 \\ d_{41} & 0 & 0 & d_{44} & d_{45} \\ d_{51} & 0 & 0 & d_{54} & d_{55} \end{bmatrix},$$

$$d_{11} = r(c_0)\Big(1-\frac{2x}{K}\Big) - \frac{ay(m-\alpha x^2)}{(\alpha x^2+m)^2}, \quad d_{12} = \frac{ax}{\alpha x^2+m}, \quad d_{13} = r_1 x\Big(1-\frac{x}{K}\Big),$$

$$d_{21} = \frac{by(m-\alpha x^2)}{(\alpha x^2+m)^2}, \quad d_{22} = d_1 + 2g_1 y + \frac{cz(h-\beta y^2)}{(\beta y+h)^2}, \quad d_{23} = \frac{cy}{\beta y^2+h},$$

$$d_{32} = \frac{dz(h-\beta y^2)}{(\beta y+h)^2}, \quad d_{33} = \frac{dy}{\beta y^2+h} - d_2 - 2g_2 z,$$

$$d_{44} = xv, \quad d_{41} = -a_2 c_e + vc_0, \quad d_{45} = -a_1 - a_2 x,$$

$$d_{51} = a_2 c_e - vc_0, \quad d_{54} = -b_1 - vx, \quad d_{55} = a_2.$$

- **At $\mathbf{E_0}$**, the eigenvalues of the characteristic equation are r_0,$-d_1$, $-d_2$ and $\pm\sqrt{a_1 b_1}$, showing the instability of E_0 since one eigenvalue is positive.
- **At $\mathbf{\bar{E}_1}$**, two eigenvalues of the characteristic equation are,$-d_1$,$-d_2$, and the remaining three eigenvalues are given by the roots of the following cubic equation

$$\lambda^3 + S_1\lambda^2 + S_2\lambda + S_3 = 0,$$

where

$$S_1 = \frac{\bar{x} r(\bar{c_0})}{K} - (a_1 + a_2\bar{x}) - r(\bar{c_0})\Big(1-\frac{\bar{x}}{K}\Big),$$
$$S_2 = c_1\bar{x}(a_2+v) + a_{13}(v\bar{c_0} - a_2\bar{c_e}) - a_2 b_1\bar{x} - a_1 b_1 - a_1 v\bar{x},$$
$$S_3 = a_{13}a_1(v\bar{c_0} - a_2\bar{c_e}) + c_1(a_2 b_1\bar{x} + a_1 b_1 + a_1 v\bar{x}),$$
$$c_1 = \frac{\bar{x} r(\bar{c_0})}{K} - (a_1 + a_2\bar{x}) - r(\bar{c_0})\Big(1-\frac{\bar{x}}{K}\Big).$$

According to Routh Hurwitz criteria $\bar{E_1}$ is locally asymptotically stable if $S_1 > 0$ and $S_1 S_2 - S_3 > 0$.
- **At $\mathbf{\hat{E}_2}$**, one of the eigenvalues of the characteristic equation is $d\hat{y}/(\beta\hat{y}^2+h) - d_2$ and the remaining four eigenvalues are given by the roots of the following equation

$$\lambda^4 + Q_1\lambda^3 + Q_2\lambda^2 + Q_3\lambda + Q_4 = 0,$$

where

$$Q_1 = d_1 + 2g_1\hat{y} - (a_2+v)\hat{x} - \frac{ab\hat{x}\hat{y}(m-\alpha\hat{x}^2)}{(\alpha x^2+m)^3} - w_1,$$

$$Q_2 = -w_1\Big[d_1 + 2g_1\hat{y} - \frac{ab\hat{x}\hat{y}(m-\alpha\hat{x}^2)}{(\alpha\hat{x}^2+m)^3}\Big] - a_1 b_1 - (a_1 v + a_2 b_1)\hat{x}$$
$$-(a_2+v)\hat{x}\Big[d_1 + 2g_1\hat{y} - \frac{ab\hat{x}\hat{y}(m-\alpha\hat{x}^2)}{(\alpha\hat{x}^2+m)^3} - w_1\Big],$$

$$Q_3 = \hat{x}(a_2+v)w_1\Big[d_1+2g_1\hat{y}-\frac{ab\hat{x}\hat{y}(m-\alpha\hat{x}^2)}{(\alpha\hat{x}^2+m)^3}\Big]-(a_1v+a_2b_1)\hat{x}$$
$$\Big[d_1+2g_1\hat{y}-\frac{ab\hat{x}\hat{y}(m-\alpha\hat{x}^2)}{(\alpha\hat{x}^2+m)^3}-w_1\Big],$$
$$Q_4 = a_1b_1+(a_1v+a_2b_1)\hat{x}-w_1\Big[d_1+2g_1\hat{y}-(a_2+v)\hat{x}-\frac{ab\hat{x}\hat{y}(m-\alpha\hat{x}^2)}{(\alpha\hat{x}^2+m)^3}\Big],$$
$$w_1 = r(\hat{c_0})\Big(1-\frac{\hat{x}}{K}\Big)+\frac{\hat{x}r(c_0)}{K}+\frac{a\hat{y}(m-\alpha\hat{x}^2)}{(\alpha\hat{x}^2+m)^2}.$$

Applying Routh–Hurwitz criteria, it is found that $\hat{E_2}$ is locally asymptotically stable if the following conditions hold:

$$\frac{d\hat{y}}{\beta\hat{y}^2+h}<d_2,$$
$$Q_1>0,\quad Q_1Q_2>Q_3,\quad Q_1Q_2Q_3>Q_3^2+Q_1^2Q_4.$$

- *The characteristic equation of* $E_3^\star$ is given as:

$$\lambda^5+R_1\lambda^4+R_2\lambda^3+R_3\lambda^2+R_4\lambda+R_5=0,$$

where

$$R_1 = -(a_{44}+a_{55}+a_{11}+a_{22}+a_{33}),$$
$$R_2 = a_{44}a_{55}-a_{51}a_{45}+(a_{44}+a_{55})(a_{22}+a_{33}+a_{11})+a_{22}a_{33}$$
$$-a_{23}a_{32}+a_{11}(a_{22}+a_{33})+a_{12}a_{21},$$
$$R_3 = -[(a_{44}a_{55}-a_{51}a_{45})(a_{22}+a_{33}+a_{11})+(a_{44}+a_{55})(a_{22}a_{33}-a_{23}a_{32}$$
$$+a_{11}(a_{22}+a_{33})+a_{12}a_{21})]+a_{13}(a_{44}a_{55}-a_{51}a_{45})+a_{41}a_{13}(a_{22}+a_{33}),$$
$$R_4 = (a_{44}a_{55}-a_{51}a_{45})(a_{22}a_{33}-a_{23}a_{32}+a_{11}(a_{22}+a_{33})+a_{12}a_{21})+$$
$$(a_{44}+a_{55})(a_{12}a_{21}a_{33}+a_{11}(a_{22}a_{33}-a_{32}a_{23})),$$
$$R_5 = -(a_{44}a_{55}-a_{51}a_{45})(a_{12}a_{21}a_{33}+a_{11}(a_{22}a_{33}-a_{32}a_{23}))-(a_{41}a_{55}-a_{51}a_{45})$$
$$(a_{13}^2a_{23}a_{32}-a_{13}a_{22}a_{33}).$$

and

$$a_{11} = r(c_0^\star)\left(1-\frac{x^\star}{K}\right)-\frac{x^\star r(c_0^\star)}{K}-\frac{ay^\star(m-\alpha x^{\star 2})}{(\alpha x^{\star 2}+m)^2},\quad a_{12}=\frac{ax^\star}{\alpha x^{\star 2}+m},$$
$$a_{13} = r_1x^\star\left(1-\frac{x^\star}{K}\right),\quad a_{21}=\frac{by^\star(m-\alpha x^{\star 2})}{(\alpha x^{\star 2}+m)^2},\quad a_{22}=d_1+2g_1y^\star+\frac{cz^\star(h-\beta y^{\star 2})}{(\beta y^\star+h)^2},$$
$$a_{23} = \frac{cy^\star}{\beta y^{\star 2}+h},\quad a_{32}=\frac{dz^\star(h-\beta y^{\star 2})}{(\beta y^\star+h)^2},\quad a_{33}=\frac{dy^\star}{\beta y^{\star 2}+h}-d_2-2g_2z^\star,$$
$$a_{41} = -a_2c_e^\star+vc_0^\star,\quad a_{44}=vx^\star,\quad a_{45}=-a_1-a_2x^\star,$$
$$a_{51} = a_2c_e^\star-vc_0^\star,\quad a_{54}=-b_1-vx^\star,\quad a_{55}=a_2x^\star.$$

According to Routh–Hurwitz criterion, the equilibrium point $E_3^\star$ is locally asymptotically stable if

$$R_1>0,\quad R_1R_2-R_3>0,\quad R_1R_2R_3>R_3^2+R_1^2R_4,\quad R_1R_2R_3+R_1R_5>R_3^2+R_1^2R_4.$$

5. Global stability

Global stability is analyzed using Lyapunov functions, ensuring that the system will settle into a steady-state solution over time.

Theorem 2. *If the following constraints are satisfied in the region* Ω :

$$r(c_0^\star)\eta_1 > K a\alpha y^\star(x_l + x^\star), \tag{5.1}$$

$$(d_1 + g_1(y_u + y^\star)) > M_4, \tag{5.2}$$

$$\eta_2(d_2 + g_2(z_u + z^\star)) > dy^\star(h - \beta y_u y^\star), \tag{5.3}$$

$$\Big(\frac{r(c_0^\star)}{K} - \frac{a\alpha y^\star(x_u + x^\star)}{\eta_1}\Big)M_1 > M_3, \tag{5.4}$$

$$M_1 M_2 \eta_2 + d(hz_u + \beta y_u y^\star z^\star) > cy^\star(h + \beta y_l y^{\star 2}), \tag{5.5}$$

$$(b + x^\star)(a_1 + a_2 x^\star) > (a_2 + v)x^\star, \tag{5.6}$$

$$(b + x^\star)\Big(\frac{r(c_0^\star)}{K} - \frac{a\alpha y^\star(x_u + x^\star)}{\eta_1}\Big) > (a_2(c_{e_l} - vc_{0_u}), \tag{5.7}$$

where

$$M_1 = (d_1 + g_1(y_u + y^\star)) - \Big(\frac{x^\star(1 + x_u\alpha b)}{\eta_1} - \frac{c(z_u h - \beta y_u y^\star z^\star)}{\eta_2}\Big),$$

$$M_2 = d_2 + g_2(z_u + z^\star) - \frac{dy^\star(h - \beta y_u y^\star)}{\eta_2},$$

$$M_3 = \Big[\frac{a(m + \alpha x^{\star 2})}{\eta_1} - \frac{b(my_u + \alpha x_u x^\star y^\star)}{\eta_2}\Big]^2,$$

$$M_4 = \Big(\frac{x^\star(1 + x_l\alpha b)}{\eta_1} - \frac{c(z_l h - \beta y_l y^\star z^\star)}{\eta_2}\Big),$$

$$\eta_1 = (\alpha x_u^2 + m)(\alpha x^{\star 2} + m), \quad \eta_2 = (\beta y_u^2 + h)(\beta y^{\star 2} + h),$$

where x_l *and* x_u, y_l *and* y_u, c_{e_l} *and* c_{0_u}, z_u *denote the lower* (l) *and upper* (u) *bounds of the respective state variables,*

$$x_l = K_4 - K_2, \quad x_u = K, \quad c_{e_l} = K_3, \quad c_{0_u} = K_2, \quad y_l = \frac{b(K_4 - K_2)}{a}, \quad y_u = K_1, \quad z_u = \frac{K_1 bd}{ac},$$

(*where values of* K_i, $i = 1, 2, 3, 4$ *can be seen at Theorem 1*) *then the positive equilibrium point* $E_3^\star$ *is globally asymptotically stable in the region* Ω.

P r o o f. We assumed the following positive definite function about $E_3^\star$:

$$L_1 = \Big(x - x^\star - x^\star \ln\Big(\frac{x}{x^\star}\Big)\Big) + \frac{n_1}{2}(y - y^\star)^2 + \frac{n_2}{2}(z - z^\star)^2 + \frac{n_3}{2}(c_e - c_e^\star)^2 + \frac{n_4}{2}(c_0 - c_0^\star)^2.$$

Differentiating L_1 with respect to time t, we get

$$\frac{dL_1}{dt} = \Big(\frac{x - x^\star}{x}\Big)\frac{dx}{dt} + n_1(y - y^\star)\frac{dy}{dt} + n_2(z - z^\star)\frac{dz}{dt} + n_3(c_e - c_e^\star)\frac{dc_e}{dt} + n_4(c_0 - c_0^\star)\frac{dc_0}{dt}.$$

After performing some algebraic manipulations using system of equations (2.1), (2.5), we obtain

$$\frac{dL_1}{dt} = -(x - x^\star)^2\Big(\frac{r(c_0^\star)}{K} - \frac{a\alpha y^\star(x + x^\star)}{\eta_1}\Big)$$

$$-(y - y^\star)^2\Big[n_1 d_1 + n_1 g_1(y + y^\star) - \Big(\frac{x^\star(1 + x\alpha b)}{\eta_1} - \frac{c(zh - \beta yy^\star z^\star)}{\eta_2}\Big)\Big]$$

$$-(z - z^\star)^2\Big[n_2(d_2 + g_2(z + z^\star)) - \frac{n_2 dy^\star(h - \beta yy^\star)}{\eta_2}\Big]$$

$$-(c_e - c_e^\star)^2 n_4(a_1 + a_2x^\star) - (c_0 - c_0^\star)^2 n_3(b + x^\star)$$
$$-(x - x^\star)(y - y^\star)\Big[\frac{a(m + \alpha x^{\star 2})}{\eta_1} - \frac{n_1 b(my + \alpha x x^\star y^\star)}{\eta_2}\Big]$$
$$-(y - y^\star)(z - z^\star)\frac{1}{\eta_2}\big(n_1 c(hy^\star + \beta y y^{\star 2}) - n_2 d(hz + \beta y y^\star z^\star)\big)$$
$$-(x - x^\star)(c_0 - c_0^\star)\Big(r_1 - \frac{r_1 x}{K} - n_3 a_2 c_e + n_3 v c_0\Big)$$
$$-(x - x^\star)(c_e - c_e^\star)n_4(a_2 c_e - v c_0) + (c_0 - c_0^\star)(c_e - c_e^\star)x^\star(a_2 + n_4 v),$$

where

$$\eta_1 = (\alpha x^2 + m)(\alpha x^{\star 2} + m), \quad \eta_2 = (\beta y^2 + h)(\beta y^{\star 2} + h).$$

Now dL_1/dt can further be written as sum of the quadratic forms as

$$\frac{dL_1}{dt} \le -\big[(b_{11}/2)(x - x^\star)^2 - b_{12}(x - x^\star)(y - y^\star) + (b_{22}/2)(y - y^\star)^2$$
$$+(b_{11}/2)(x - x^\star)^2 + b_{14}(x - x^\star)(c_e - c_e^\star) + (b_{44}/2)(c_e - c_e^\star)^2$$
$$+(b_{11}/2)(x - x^\star)^2 - b_{15}(x - x^\star)(c_0 - c_0^\star) + (b_{55}/2)(c_0 - c_0^\star)^2$$
$$+(b_{22}/2)(y - y^\star)^2 + b_{23}(y - y^\star)(z - z^\star) + (b_{33}/2)(z - z^\star)$$
$$+(b_{44}/2)(c_e - c_e^\star)^2 - b_{45}(c_e - c_e^\star)(c_0 - c_0^\star) + (b_{55}/2)(c_0 - c_0^\star)^2\big],$$

where

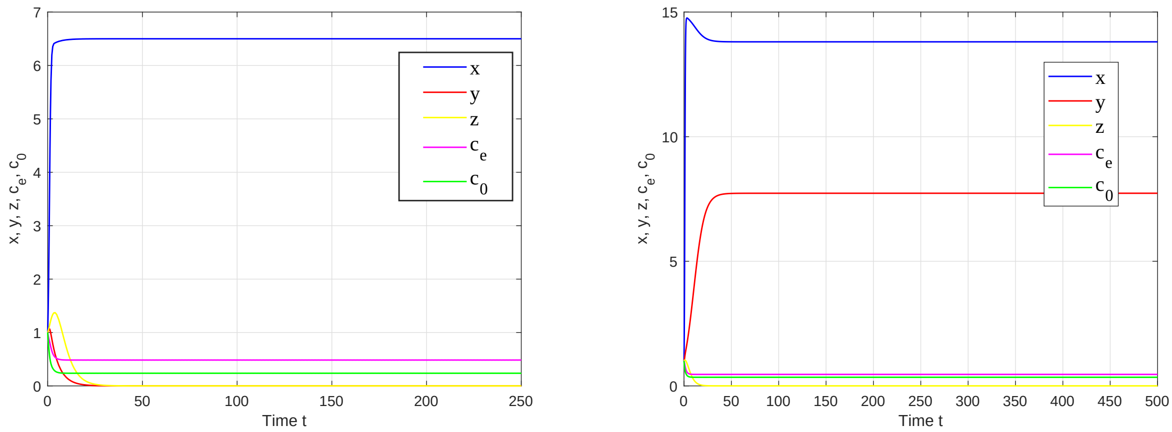
$$b_{11} = \frac{r(c_0^\star)}{K} - \frac{a\alpha y^\star(x + x^\star)}{\eta_1}, \quad b_{22} = n_1 d_1 + n_1 g_1(y + y^\star) - \left(\frac{x^\star(1 + x\alpha b)}{\eta_1} - \frac{c(zh - \beta y y^\star z^\star)}{\eta_2}\right),$$
$$b_{33} = n_2(d_2 + g_2(z + z^\star)) - \frac{n_2 d y^\star(h - \beta y y^\star)}{\eta_2}, \quad b_{44} = n_4(a_1 + a_2 x^\star), \quad b_{55} = n_3(b + x^\star),$$
$$b_{12} = \frac{a(m + \alpha x^{\star 2})}{\eta_1} - \frac{n_1 b(my + \alpha x x^\star y^\star)}{\eta_2}, \quad b_{23} = \frac{1}{\eta_2}(n_1 c(hy^\star + \beta y y^{\star 2}) - n_2 d(hz + \beta y y^\star z^\star)),$$
$$b_{45} = x^\star(a_2 + n_4 v), \quad b_{15} = (r_1 - \frac{r_1 x}{K} - n_3 a_2 c_e + n_3 v c_0).$$

Now, by using Sylvesters criteria and by choosing

$$n_1 = \frac{a(m + \alpha x^{\star 2})\eta_2}{\eta_1 b(my + \alpha x x^\star y^\star)} > 0$$

and $n_2 = n_3 = n_4 = 1$ we get dL_1/dt is negative definite under the following conditions:

$$b_{11} > 0, \tag{5.8}$$
$$b_{22} > 0, \tag{5.9}$$
$$b_{33} > 0, \tag{5.10}$$
$$b_{11}b_{22} > b_{12}^2, \tag{5.11}$$
$$b_{11}b_{44} > b_{14}^2, \tag{5.12}$$
$$b_{22}b_{33} > b_{23}^2, \tag{5.13}$$
$$b_{11}b_{55} > b_{15}^2, \tag{5.14}$$
$$b_{44}b_{55} > b_{45}^2. \tag{5.15}$$



(a) Stable graph around the equilibrium point $\bar{E}_1$.

(b) Stable graph around the equilibrium point $\hat{E}_2$.

Figure 2. Stable graph around the equilibrium points $\bar{E}_1$ and $\hat{E}_2$

It is observed that the fourth inequality, *i.e.*, $b_{11}b_{22} > b_{12}^2$ is satisfied due to the proper choice of n_1, and for other inequalities, (5.1) $\Rightarrow$ (5.8), (5.2) $\Rightarrow$ (5.9), (5.3) $\Rightarrow$ (5.10), (5.4) $\Rightarrow$ (5.12), (5.5) $\Rightarrow$ (5.13), (5.6) $\Rightarrow$ (5.14), (5.7) $\Rightarrow$ (5.15). Hence L_1 is a Lyapunov function with respect to $E_3^\star$, whose domain contains the region of attraction Ω, which proves the theorem. $\square$

6. Simulations and discussion

In this section, we numerically explore the effects of key parameters on population interaction using MATLAB and MATHEMATICA software.

We have taken the following parameter values for $\bar{E}_1$:

$$r_0 = 3.05, \quad r_1 = 0.75, \quad K = 6.5, \quad a = 1.12, \quad \alpha = 0.49, \quad m = 1.48, \quad c = 0.01,$$
$$b = 1.21, \quad d_1 = 0.571, \quad g_1 = 0.02, \quad d = 3.1, \quad \beta = 1.42, \quad h = 7, \quad d_2 = 0.223,$$
$$g_2 = 0.025, \quad q_0 = 0.515, \quad v = 0.21, \quad a_1 = 0.81, \quad a_2 = 0.142, \quad b_1 = 0.52.$$

It has been found that under the above set of parameters, the equilibrium point $\bar{E}_1$ is locally asymptotically stable (see Fig. 2a).

$$\bar{x} = 6.5, \quad \bar{y} = 0, \quad z = 0, \quad \bar{c_e} = 0.4837, \quad \bar{c_0} = 0.2368.$$

We select the following parameter values for the equilibrium $\hat{E}_2$:

$$r_0 = 3.65, \quad r_1 = 0.52, \quad K = 15, \quad a = 1.99, \quad \alpha = 0.25, \quad m = 8.0458, \quad c = 0.01,$$
$$b = 1.01, \quad d_1 = 0.0571, \quad g_1 = 0.025, \quad d = 1.0571, \quad \beta = 2.192, \quad h = 0.1568, \quad d_2 = 0.35,$$
$$g_2 = 0.0351, \quad q_0 = 0.515, \quad v = 0.821, \quad a_1 = 0.92881, \quad a_2 = 0.63, \quad b_1 = 0.252.$$

It has been observed that under the above set of parameters, the equilibrium point $\hat{E}_2$ is locally asymptotically stable (see Fig. 2b).

$$\hat{x} = 13.85, \quad \hat{y} = 7.4350, \quad z = 0, \quad \hat{c_e} = 0.4611, \quad \hat{c_0} = 0.3453.$$

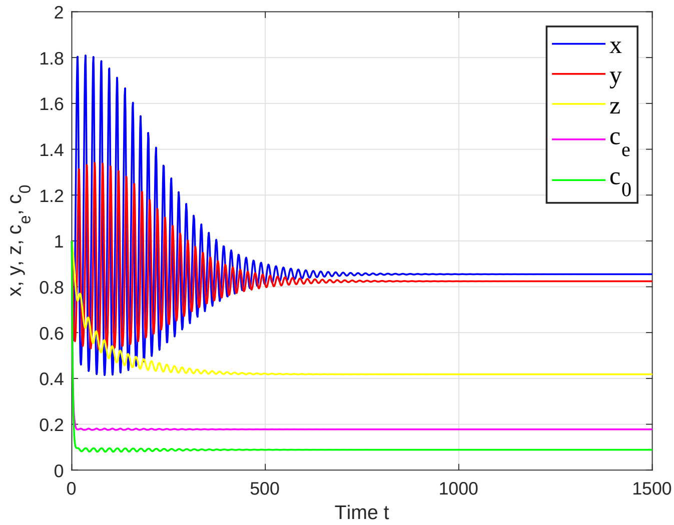


Figure 3. Stable graph around the equilibrium point $E_3^\star$.

We choose the following parameter values for $E_3^\star$:

$$r_0 = 0.58, \quad r_1 = 0.26, \quad K = 10, \quad a = 2.891, \quad \alpha = 0.653, \quad m = 4.2, \quad c = 0.671,$$
$$b = 1.46, \quad d_1 = 0.171, \quad g_1 = 0.085, \quad d = 0.59, \quad \beta = 0.52, \quad h = 10.53, \quad d_2 = 0.03,$$
$$g_2 = 0.0351, \quad q_0 = 0.155, \quad v = 0.8421, \quad a_1 = 0.81, \quad a_2 = 0.492, \quad b_1 = 0.1252.$$

It has been found that under the above set of parameters, the equilibrium point $E_3^\star$ is locally asymptotically stable (see Fig. 3 and Fig. 4).

$$x^\star = 0.7446, \quad y^\star = 0.9126, \quad z = 0.5445, \quad c_e^\star = 0.1780, \quad c_0^\star = 0.08689.$$

The bifurcation diagrams of phytoplankton, zooplankton, and fish with respect to K are presented in Fig. 5 and Fig. 6, where

$$r_0 = 0.58, \quad r_1 = 0.26, \quad a = 2.891, \quad \alpha = 0.653, \quad m = 4.2, \quad c = 0.671,$$
$$b = 1.46, \quad d_1 = 0.171, \quad g_1 = 0.085, \quad d = 0.59, \quad \beta = 0.52, \quad h = 10.53, \quad d_2 = 0.03,$$
$$g_2 = 0.0351, \quad q_0 = 0.155, \quad v = 0.8421, \quad a_1 = 0.81, \quad a_2 = 0.492, \quad b_1 = 0.1252.$$

For the above set of parameter values, we observed that if we change K from $6 \leq K \leq 7.5$ the system remains stable but shows oscillatory behavior in $7.55 \leq K \leq 10$.

Again, let us choose the following parameters

$$r_0 = 3.28, \quad K = 10, \quad a = 12.891, \quad \alpha = 0.0653, \quad m = 4.2, \quad c = 9.8671,$$
$$b = 11.46, \quad d_1 = 0.9971, \quad g_1 = 0.07685, \quad d = 5.59, \quad \beta = 2.952, \quad h = 10.53, \quad d_2 = 0.39,$$
$$g_2 = 0.015351, \quad q_0 = 0.151, \quad v = 0.8421, \quad a_1 = 0.81, \quad a_2 = 0.493, \quad b_1 = 0.1252.$$

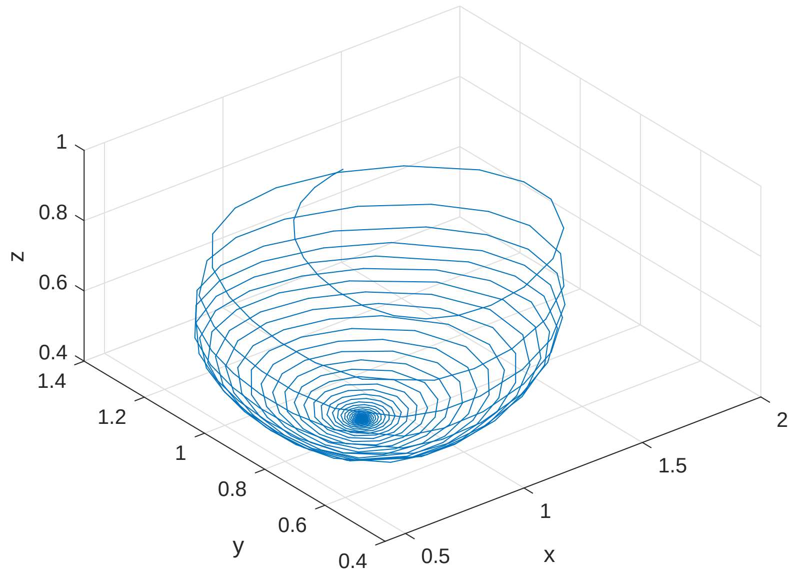


Figure 4. Phase graph around the equilibrium point $E_3^{\star}$.

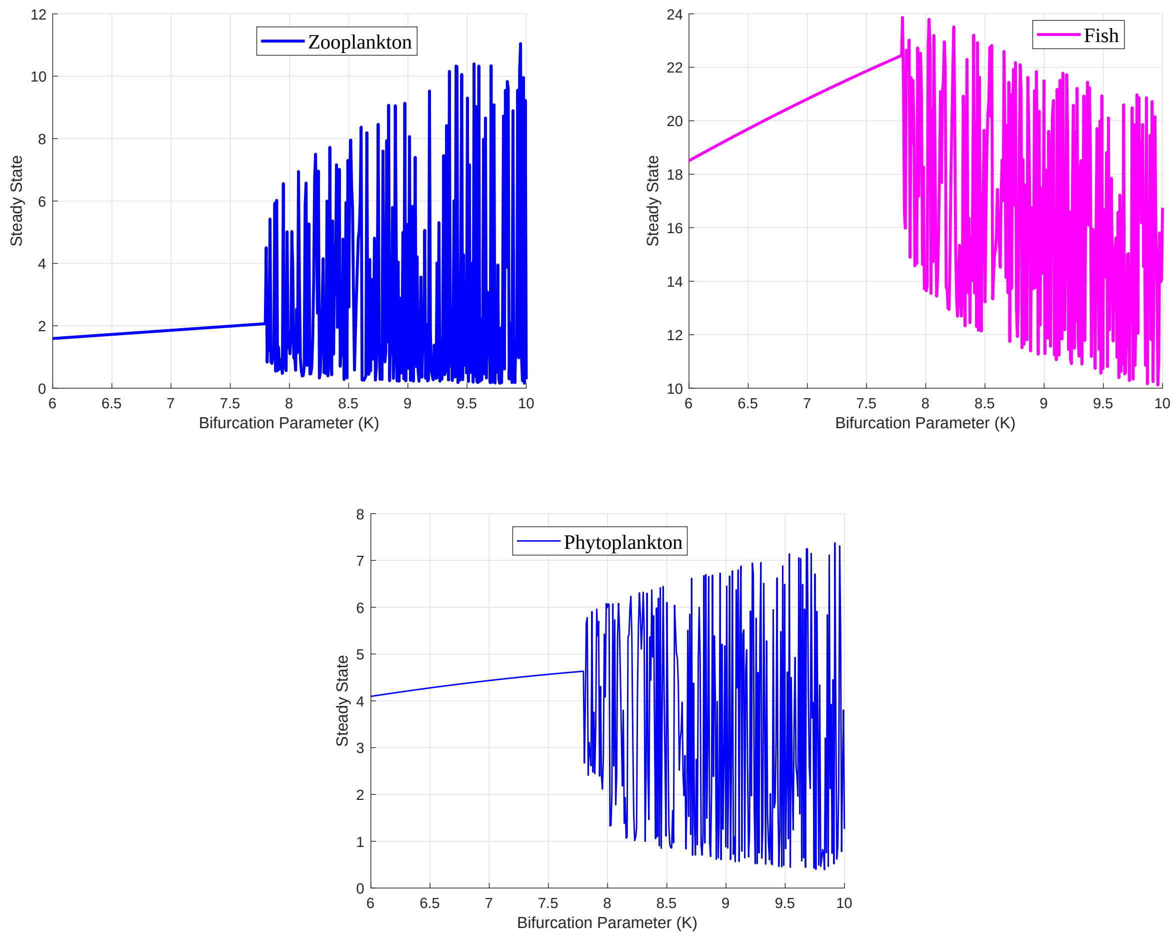


Figure 5. Bifurcation diagram of the model with respect to K.

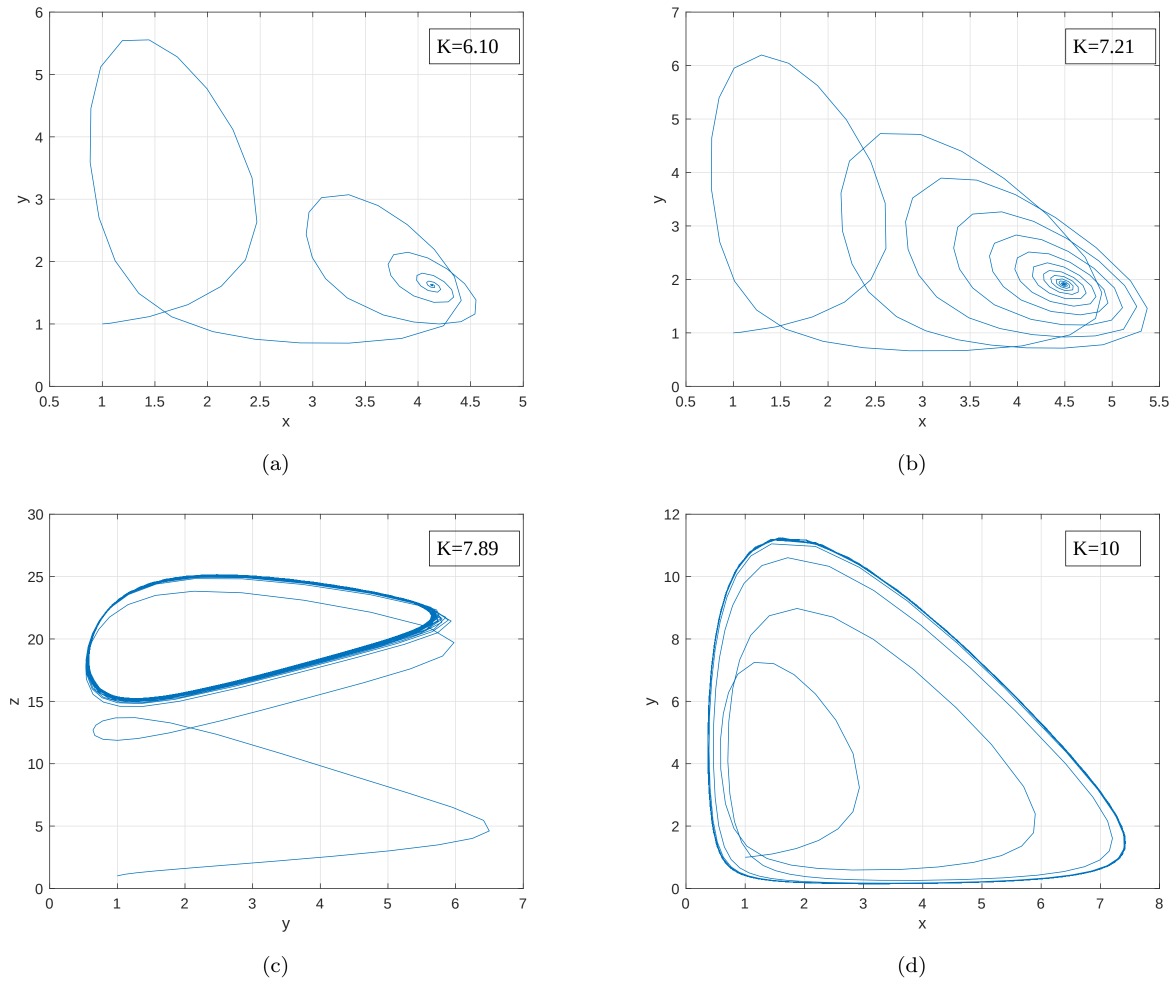


Figure 6. Phase graph of the system for different values of K.

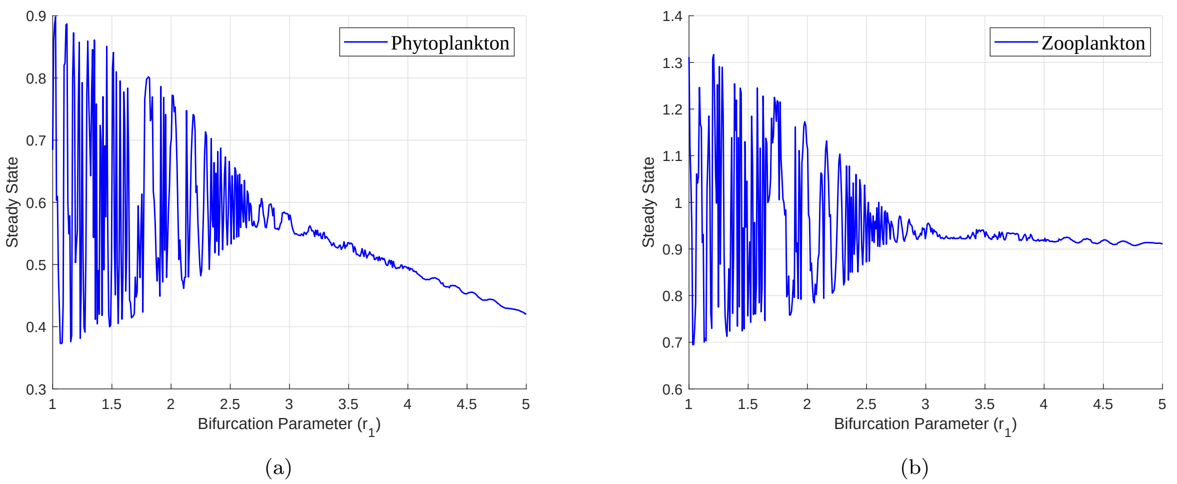


Figure 7. Bifurcation diagram of the system with respect to different values of r_1.

Bifurcation diagrams of phytoplankton and zooplankton with respect to r_1 are presented in Fig. 7a and 7b. Phase graphs for different values of r_1 showing limit cycle behavior are given at Fig. 8.

For the above set of parameter values, we observed that if we change r_1 from $1 \leq r_1 \leq 2.55$ the system shows oscillatory behavior, but is stable in $2.55 \leq r_1 \leq 10$.

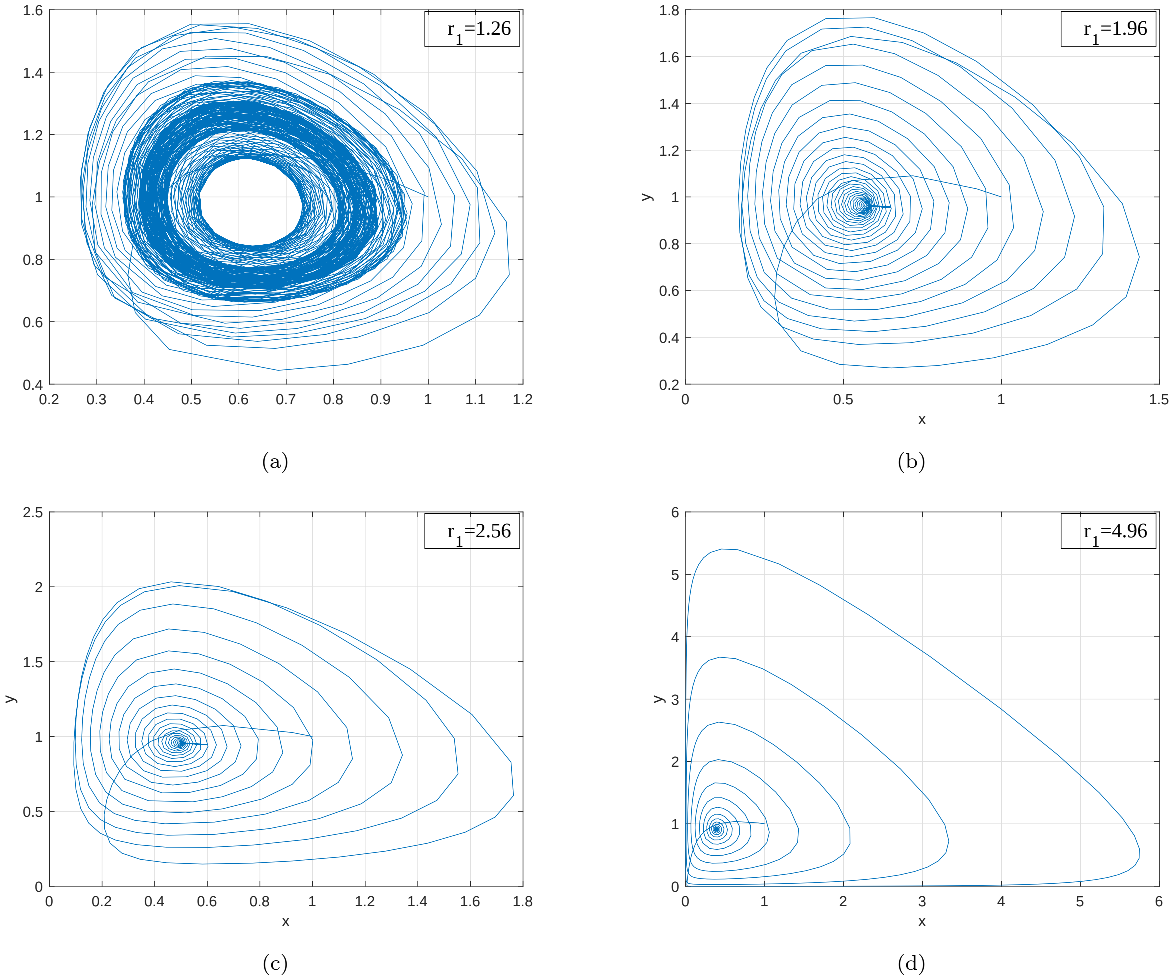


Figure 8. Phase graph of the system with respect to different values of r_1.

7. Conclusion

In this study, we proposed a mathematical model to explore the impact of toxicants in a tri-trophic marine food chain system. We established the boundedness of the system, which ensures that the population of the species remains within the feasible region. The local stability of the equilibrium point in the model has been analyzed using the Jacobian matrix. From the stability of $\bar{E}_1$, it can be concluded that the only population of phytoplankton will survive, and the population of zooplankton and fish would tend to go extinct (see Fig. 2a). The stability of $\hat{E}_2$ indicates that the phytoplankton and zooplankton population will survive and the fish will extinct (see Fig. 2b). The interior equilibrium point $E_3^\star$ is locally and globally stable, showing coexistence of all three populations (see Fig. 3). From this analysis, it is seen that some parameter associated with our proposed model can make the system unstable. Our investigation shows that a few parameters related to our suggested model have the potential to cause system instability. The numerical simulation indicates that increasing the system's carrying capacity K keeps it stable up to a critical value, after which it becomes unstable (Fig. 5). Also, it is concluded that r_1 has a significant role in the stability of the ecosystem (Fig. 7). Phase portraits are also presented, which show the limit cycle behavior of the system for different values of the parameters.

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