NOTE ON SUPER $(a, 1) - P_3$-ANTIMAGIC TOTAL LABELING OF STAR $S_n$

S. Rajkumar†, M. Nalliah‡‡ and Madhu Venkataraman†††

Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology Vellore-632 014, India
†raj26101993@gmail.com, ‡‡nalliahklu@gmail.com, †††madhu.v@vit.ac.in

Abstract: Let $G = (V, E)$ be a simple graph and $H$ be a subgraph of $G$. Then $G$ admits an $H$-covering, if every edge in $E(G)$ belongs to at least one subgraph of $G$ that is isomorphic to $H$. An $(a, d) - H$-antimagic total labeling of $G$ is bijection $f: V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, |V(G)| + |E(G)|\}$ such that for all subgraphs $H'$ of $G$ isomorphic to $H$, the $H'$-weights $w(H') = \sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e)$ constitute an arithmetic progression $(a, a + d, a + 2d, \ldots, a + (n - 1)d)$, where $a$ and $d$ are positive integers and $n$ is the number of subgraphs of $G$ isomorphic to $H$. The labeling $f$ is called a super $(a, d) - H$-antimagic total labeling if $f(V(G)) = \{1, 2, 3, \ldots, |V(G)|\}$. In [5], David Laurence and Kathiresan posed a problem that characterizes the super $(a, 1) - P_3$-antimagic total labeling of Star $S_n$, where $n = 6, 7, 8, 9$. In this paper, we completely solved this problem.

Keywords: $H$-covering, Super $(a, d) - H$-antimagic, Star.

1. Introduction

Let $G = (V(G), E(G))$ and $H = (V(H), E(H))$ be simple and finite graphs. Let $|V(G)| = v_G$, $|E(G)| = e_G$, $|V(H)| = v_H$ and $|E(H)| = e_H$. An edge covering of $G$ is a family of different subgraphs $H_1, H_2, H_3, \ldots, H_k$ such that any edge of $E(G)$ belongs to at least one of the subgraphs $H_j$, $1 \leq j \leq k$. If the $H_j$'s are isomorphic to a given graph $H$, then $G$ admits an $H$-covering. Gutin and Lladó [2] defined $H$-magic labeling, which is a generalization of Kotzig and Rosa’s edge magic total labeling [4]. A bijection $f: V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, v_G + e_G\}$ is called an $H$-magic labeling of $G$ if there exists a positive integer $k$ such that each subgraph $H'$ of $G$ isomorphic to $H$ satisfies

$$w(H') = \sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e) = k.$$ 

In this case, they say that $G$ is $H$-magic. When $f(V(G)) = \{1, 2, 3, \ldots, v_G\}$, we say that $G$ is $H$-super magic. On the other hand, Inayah et al. [3] introduced $(a, d) - H$-antimagic total labeling of $G$ which is defined as a bijection $f: V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, v_G + e_G\}$ such that for all subgraphs $H'$ of $G$ isomorphic to $H$, the set of $H'$-weights

$$w(H') = \sum_{v \in V(H')} f(v) + \sum_{e \in E(H')} f(e)$$

constitutes an arithmetic progression $a, a + d, a + 2d, \ldots, a + (n - 1)d$, where $a$ and $d$ are some positive integers and $n$ is the number of subgraphs isomorphic to $H$. In this case, they say that $G$ is $(a, d) - H$-antimagic. If $f(V(G)) = \{1, 2, 3, \ldots, v_G\}$, they say that $f$ is a super $(a, d) - H$-antimagic total labeling and $G$ is super $(a, d) - H$-antimagic. This labeling is a more general case of super $(a, d)$-edge-antimagic total labelings. If $H \cong K_2$, then we say that super $(a, d) - H$-antimagic
labelings, which is also called super \((a, d)\)-edge-antimagic total labelings and have been introduced in [6]. They studied some basic properties of such labeling and also proved the following theorem.

**Theorem 1** [3]. If \(G\) has a super \((a, d) - H\)-antimagic total labeling and \(t\) is the number of subgraphs of \(G\) isomorphic to \(H\), then \(G\) has a super \((a', d) - H\)-antimagic total labeling, where 
\[a' = [(v_G + 1)v_H + (2v_G + e_G + 1)e_H] - a - (t - 1)d.\]

Several authors are studied antimagic type labeling of graphs see [1]. In 2015, Laurence and Kathiresan [5] obtained an upper bound of \(d\) for any graph \(G\), and they investigated the existence of super \((a, d) - P_3\)-antimagic total labeling of star graph \(S_n\). First, they observed that \(S_n\) admits a \(P_h\)-covering for \(h = 2, 3\), and the star \(S_n\) contains

\[t = \binom{n}{h - 1}\]

subgraphs \(P_h\), \(h = 2, 3\), which is denoted by \(P_{h}^j\), \(1 \leq j \leq h\). In 2005, Sugeng et al. [7] investigated the case \(h = 2\) using super \((a, d)\)-edge-antimagic total labeling. In 2015, the case of \(h = 3\) was investigated by Laurence and Kathiresan [5]. Here they observed that if the star \(S_n, n \geq 3\) admits a super \((a, d) - P_3\)-antimagic total labeling then \(d \in \{0, 1, 2\}\). Now, they proved the star \(S_n, n \geq 3\) has super \((4n + 7, 0) - P_3\)-antimagic total labeling and \(S_n, n \geq 3\) admits a super \((a, 2) - P_3\)-antimagic total labeling if and only if \(n = 3\). Also, they proved the following theorems and posed a problem.

**Theorem 2** [5]. If the star \(S_n, n \geq 3\) has super \((a, 1) - P_3\)-antimagic total labeling, then \(3 \leq n \leq 9\). Moreover, the star \(S_n\) admits a super \((a, 1) - P_3\)-antimagic total labeling, where \(a = 19\), for \(n = 3\) and \(a = 21\), for \(n = 4\).

**Theorem 3** [5]. For \(n = 5\), the star \(S_n\) has no super \((a, 1) - P_3\)-antimagic total labeling.

**Problem 1.** [5] For each \(n, 6 \leq n \leq 9\) characterize the super \((a, 1) - P_3\)-antimagic total labeling for the star \(S_n\).

In this paper, we present the complete solution to the above problem.

### 2. Main Results

Let \(S_n \cong K_{1,n}, n \geq 1\) be the star graph and let \(v_0\) be the central vertex and let \(v_i, 1 \leq i \leq n\) be its adjacent vertices. Thus \(S_n\) has \(n + 1\) vertices and \(n\) edges.

**Theorem 4.** The star \(S_6\) has no super \((a, 1) - P_3\)-antimagic total labeling.

**Proof.** Let \(V(S_6) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6\}\) and \(E(S_6) = \{v_0v_1, v_0v_2, v_0v_3, v_0v_4, v_0v_5, v_0v_6\}\) be the vertex set and the edge set of Star \(S_6\). Suppose there exists a super \((a, 1) - P_3\)-antimagic total labeling \(f : V \cup E \to \{1, 2, 3, \ldots, 13\}\) for \(S_6\) and let \(v_0\) be the central vertex of \(S_6\). In the computation of \(P_3\) — weights the label of the central vertex \(v_0, f(v_0)\) is used 15 times and label of other vertices and edges say \(i\) are used 5 times each. Therefore,

\[10f(v_0) + 5 \sum_{i=1}^{13} (i) = \frac{15}{2}(2a + 14),\]

which implies \(a = (70 + 2f(v_0))/3\). Since \(1 \leq f(v_0) \leq 7\), it follows that \(a = 24\) if \(f(v_0) = 1\), \(a = 26\) if \(f(v_0) = 4\) and \(a = 28\) if \(f(v_0) = 7\).
Case (i): $f(v_0) = 1$. Then $a = 24$ and the $P_3$ — weights of $S_6$ are given by $W = \{24, 25, \ldots, 38\}$. Now, the $P_3$ — weight 24 is getting exactly two possible 5 elements sum $(1, 2, 4, 8, 9)$ or $(1, 2, 3, 8, 10)$ and hence the label of edges $e_1 = v_0v_1$ and $e_2 = v_0v_3$ or $v_0v_2$ is $f(e_1) = 8$ and $f(e_2) = 9$ or 10.

Subcase (i): $f(e_2 = v_0v_3) = 9$. Then $a = 24$ and hence the label of the vertices and edges are $f(v_0) = 1, f(v_1) = 2, f(v_2) = 4, f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_3) = 9$. Now, the $P_3$ — weight 25 is getting exactly one possible 5 elements sum $(1, 2, 3, 8, 11)$ and hence the label of an edge $e_3 = v_0v_2$ is $f(e_3) = 11$. Also, the $P_3$ — weight 26 is getting exactly one possible 5 elements sum $(1, 2, 5, 8, 10)$ and hence the label of an edge $e_4 = v_0v_4$ is $f(e_4) = 10$.

Let $x = v_0v_5$ and $y = v_0v_6$ be two edges of $S_6$ (see Fig. 1). Clearly, the label of the edges $x$ and $y$ is $f(x) = 12$ or 13 and $f(y) = 13$ or 12. If $f(x) = 12$ then $f(y) = 13$ and hence there is no $P_3$ — weight 27. Also, if $f(x) = 13$ then $f(y) = 12$ and hence there is no $P_3$ — weight 27, which is a contradiction.

A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ with $f(e_1) = 9$ and $f(e_2) = 8$ for the $P_3$ — weight 24 is used to getting the $P_3$ — weight 27.

Subcase (ii): $f(e_2 = v_0v_2) = 10$. Then $a = 24$ and hence the label of the vertices and edges of $P_3$ — weight 24 is $f(v_0) = 1, f(v_1) = 2, f(v_2) = 3, f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_2) = 10$. Now, the $P_3$ — weight 25 is getting exactly one possible 5 elements sum $(1, 2, 5, 8, 9)$ and hence the label of an edge $e_3 = v_0v_4$ is $f(e_3) = 9$. Also, the $P_3$ — weight 26 is getting exactly one possible 5 elements sum $(1, 2, 4, 8, 11)$ and hence the label of an edge $e_4 = v_0v_3$ is $f(e_4) = 11$. Let $x = v_0v_5$ and $y = v_0v_6$ be two edges of $S_6$ (see Fig. 2). Clearly, the label of the edges $x$ and $y$ is $f(x) = 12$ or 13 and $f(y) = 13$ or 12. If $f(x) = 12$ then $f(y) = 13$ and hence there is no $P_3$ — weight 27. Also, If $f(x) = 13$ then $f(y) = 12$ and hence there is no $P_3$ — weight 27, which is a contradiction.

A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ with $f(e_1) = 10$ and $f(e_2) = 8$ for the $P_3$ — weight 24 is used to getting the $P_3$ — weight 27.

Case (ii): $f(v_0) = 7$. Then $a = 28$. Now, if $f$ is a super $(28, 1) - P_3$-antimagic total labeling of $S_6$, then by Theorem 1 [3], $f$ is a super $(24, 1) - P_3$-antimagic total labeling, which does not exist by Case (i).

Case (iii): $f(v_0) = 4$. Then $a = 26$ and hence the $P_3$ — weights of $S_6$ are given by $W = \{26, 27, \ldots, 40\}$. Now, the $P_3$ — weight 26 is getting exactly four possibles 5 elements sum such as $(4, 1, 2, 8, 11)$, $(4, 1, 2, 9, 10)$, $(4, 2, 3, 8, 9)$ and $(4, 1, 3, 8, 10)$ and hence the edges $e_1 = v_0v_1$ or $v_0v_2$ and $e_2 = v_0v_2$ or $v_0v_3$ with $f(e_1) = 8$ or 9 and $f(e_2) = 9$ or 10 or 11.

Subcase (i): $f(e_1 = v_0v_1) = 8$ and $f(e_2 = v_0v_2) = 11$. Then $a = 26$ and hence the label of the vertices and edges of $P_3$ — weight 26 is $f(v_0) = 4, f(v_1) = 1, f(v_2) = 2, f(e_1 = v_0v_1) = 8$ and

![Figure 1](image-url)
Note on Super \((a, 1) - P_3\)-antimagic Total Labeling of Star \(S_n\)

\[ \begin{array}{c}
1 \\
2 \quad 3 \\
4 \quad 5 \\
6 \quad 7
\end{array} \]

Figure 2. The possible edge labels \(x\) and \(y\) are obtained \(P_3\)-weight 27.

\[ \begin{array}{c}
4 \\
8 \\
11 \\
12 \\
9 \\
10 \\
13 \\
1 \quad 2 \\
3 \\
5 \\
6 \\
7
\end{array} \]

Figure 3. There is no possible to obtain \(P_3\)-weight 30.

\(f(e_2 = v_0v_2) = 11\). Now, the \(P_3\) — weight 27, 28 and 29 are getting exactly one possible 5 elements sum \((4, 1, 5, 8, 9), (4, 1, 3, 8, 12)\) and \((4, 1, 6, 8, 10)\). Hence the label of the edges \(e_3 = v_0v_3, e_4 = v_0v_4, e_5 = v_0v_5\) and \(e_6 = v_0v_6\) is \(f(e_3) = 12, f(e_4) = 9, f(e_5) = 10\) and \(f(e_6) = 13\). From Fig. 3, there is no \(P_3\) — weight is 30, which is a contradiction.

A similar contradiction arises, if the edges \(e_1\) and \(e_2\) with \(f(e_1 = v_0v_1) = 11\) and \(f(e_2 = v_0v_2) = 8\) for \(P_3\) — weight 26 are used to getting the \(P_3\) — weight 33, for more details see Fig. 4.

**Subcase (ii):** \(f(e_1 = v_0v_1) = 9\) and \(f(e_2 = v_0v_2) = 10\). Then \(a = 26\) and hence the label of the vertices and edges of \(P_3\) — weight 26 is \(f(v_0) = 4, f(v_1) = 1, f(v_2) = 2, f(e_1 = v_0v_1) = 9\) and \(f(e_2 = v_0v_2) = 10\). Now, the \(P_3\) — weight 27 is getting exactly two possibilities 5 elements sum as \((4, 2, 3, 10, 8), (4, 1, 5, 9, 8)\) and hence the label of the edges \(e_3 = v_0v_3\) or \(v_0v_4\) is \(f(e_3) = 8\). If an edge \(e_3 = v_0v_3\) with \(f(e_3) = 8\) then we get the \(P_3\) — weight as sum of 5 elements \((4, 1, 3, 9, 8)\) is 25, which is a contradiction. If an edge \(e_3 = v_0v_4\) with \(f(e_3) = 8\) then we get the \(P_3\) — weights from 28 to 32 are getting exactly one possible 5 elements sum such as \((4, 1, 3, 9, 11), (4, 2, 5, 10, 8), (4, 2, 3, 10, 11), (4, 3, 5, 11, 8)\) and \((4, 1, 6, 9, 12)\). From Fig. 5, there is no \(P_3\) — weight 33, which is a contradiction.

A similar contradiction arises, if the edges \(e_1 = v_0v_1\) and \(e_2 = v_0v_2\) with \(f(e_1 = v_0v_1) = 10\) and \(f(e_2 = v_0v_2) = 9\) for the \(P_3\) — weight 26 is used to getting the \(P_3\) — weight 27, which is a contradiction.

**Subcase (iii):** \(f(e_1 = v_0v_2) = 8\) and \(f(e_2 = v_0v_3) = 9\). Then \(a = 26\) and hence the label of the vertices and edges of \(P_3\) — weight 26 is \(f(v_0) = 4, f(v_2) = 2, f(v_3) = 3, f(e_1 = v_0v_2) = 8\) and \(f(e_2 = v_0v_3) = 9\). Now, the \(P_3\) — weight 27 is getting exactly one possible 5 elements sum \((4, 1, 3, 9, 10)\) and hence the label of an edge \(e_3 = v_0v_1\) is \(f(e_3) = 10\). Thus, we get a \(P_3\) — weight
as sum of 5 elements \((4, 1, 2, 10, 8)\) is 25, which is a contradiction.

A similar contradiction arises, if the edges \(e_1 = v_0v_2\) and \(e_2 = v_0v_3\) with \(f(e_1 = v_0v_2) = 9\) and \(f(e_2 = v_0v_3) = 8\) for the \(P_3\) — weight 26. The \(P_3\) — weight 27 is getting exactly one possible 5 elements sum \((4, 1, 2, 11, 9)\) and hence the label of an edge \(f(e_3 = v_0v_1) = 11\). Thus, we get the \(P_3 = (v_0, v_1, v_3, e_3 = v_0v_1, e_2 = v_0v_3)\) with weight \((4 + 1 + 3 + 11 + 8)\) is 27, which is a contradiction.

**Subcase (iv):** \(f(e_1 = v_0v_2) = 8\) and \(f(e_2 = v_0v_3) = 10\). Then \(a = 26\) and hence the label of the vertices and edges of \(P_3\) — weight 26 is \(f(v_0) = 4, f(v_1) = 1, f(v_3) = 3, f(e_1 = v_0v_1) = 8\) and \(f(e_2 = v_0v_3) = 10\). Now, the \(P_3\) — weight 27 is getting exactly two possible 5 elements sum such as \((4, 1, 2, 8, 12), (4, 1, 5, 8, 9)\) and hence the label of the edges \(e_3 = v_0v_2\) or \(v_0v_4\) is \(f(e_3) = 12\) or 9. If an edge \(e_3 = v_0v_2\) with \(f(e_3) = 12\) then the \(P_3\) — weights 28 and 29 are getting exactly one possible 5 elements sum \((4, 1, 6, 8, 9)\) and \((4, 1, 5, 8, 11)\). From Fig. 6, there is no \(P_3\) — weight 30, which is a contradiction. If an edge \(e_4 = v_0v_4\) with \(f(e_4) = 9\) then the \(P_3\) — weight 28 is getting exactly one possible 5 elements sum \((4, 1, 2, 8, 13)\) and hence the label of an edge \(e_5 = v_0v_2\) is \(f(e_5) = 13\). From Fig. 7, there is no \(P_3\) — weight 29 when \(x = 11\) or 12 and \(y = 12\) or 11, which is a contradiction.

A similar contradiction arises, if the edges \(e_1 = v_0v_1\) and \(e_2 = v_0v_3\) with \(f(e_1 = v_0v_1) = 10\) and \(f(e_2 = v_0v_3) = 8\) for the \(P_3\) — weight 26 are used to getting the \(P_3\) — weight 27, which is a contradiction. \(\square\)

**Theorem 5.** *The star \(S_7\) has no super \((a, 1) - P_3\)-antimagic total labeling.*

**Proof.** Let \(V(S_7) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7\}\) and \(E(S_7) = \{v_0v_1, v_0v_2, v_0v_3, v_0v_4, v_0v_5, v_0v_6, v_0v_7\}\) be the vertex and edge set of star \(S_7\). Suppose there exists a super \((a, 1) - P_3\)-antimagic total labeling \(f : V \cup E \to \{1, 2, 3, \ldots, 15\}\) for \(S_7\) and let \(v_0\) be the central vertex of \(S_7\). In the
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Figure 6. There is no possible to obtain \(P_3\)-weight 30.

Figure 7. There is no possible to obtain \(P_3\)-weight 29.

computation of \(P_3\) — weights the label of the central vertex \(v_0\), \(f(v_0)\) is used 21 times and label of other vertices and edges say \(i\) are used 6 times each. Therefore,

\[ 15f(v_0) + 6 \sum_{i=1}^{15} (i) = \frac{21}{2}[2a + 20], \]

which implies that we get

\[ a = \frac{15f(v_0) + 510}{21}. \]

Since \(1 \leq f(v_0) \leq 8\), we have only two values \(a\) such as \(a = 25\) if \(f(v_0) = 1\) and \(a = 30\) if \(f(v_0) = 8\).

Case (i): \(f(v_0) = 1\). Then \(a = 25\) and the \(P_3\) — weights of \(S_7\) is given by \(W = \{25, 26, \ldots, 45\}\). Now, the \(P_3\) — weight 25 is getting exactly one possible 5 elements sum \((1, 2, 3, 9, 10)\) and hence the label of edges \(e_1 = v_0v_1\) and \(e_2 = v_0v_2\) is \(f(e_1) = 9\) and \(f(e_2) = 10\). Since the minimum possible sum of vertices labels for \(P_3\) — weight is 7, it follows that there is no \(P_3\) — weight 26, which is a contradiction. A similar contradiction arises, if the edges \(e_1 = v_0v_1\) and \(e_2 = v_0v_2\) with \(f(e_1) = 10\) and \(f(e_2) = 9\) for the \(P_3\) — weight 25 is used to getting the \(P_3\) — weight 27.

Case (ii): \(f(v_0) = 8\). Then \(a = 30\). Now, if \(f\) is a super \((30, 1) - P_3\)-antimagic total labeling of \(S_6\), then by Theorem 1 \([3]\), \(\hat{f}\) is a super \((25, 1) - P_3\)-antimagic total labeling, which does not exist by Case (i). \(\square\)

Theorem 6. The star \(S_8\) has no super \((a, 1) - P_3\)-antimagic total labeling.

Proof. Let \(V(S_8) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}\) and \(E(S_8) = \{v_0v_1, v_0v_2, v_0v_3, v_0v_4, v_0v_5, v_0v_6, v_0v_7, v_0v_8\}\) be the vertex and edge set of star \(S_8\). Suppose there exists a super \((a, 1) - P_3\)-antimagic total labeling \(f : V \cup E \rightarrow \{1, 2, 3, \ldots, 17\}\) for \(S_8\) and let \(v_0\) be the central vertex of \(S_8\).
In the computation of $P_3$ — weights the label of the central vertex $v_0$, $f(v_0)$ is used 28 times and label of other vertices and edges say $i$ are used 7 times each. Therefore,

$$21f(v_0) + 7 \sum_{i=1}^{17}(i) = \frac{28}{2}[2a + 27],$$

which implies that we get

$$a = \frac{21f(v_0) + 693}{28}.$$ 

Since $1 \leq f(v_0) \leq 9$, we have only two values $a$ such as $a = 27$, if $f(v_0) = 3$ and $a = 30$, if $f(v_0) = 7$.

**Case (i):** $f(v_0) = 3$. Then $a = 27$ and the $P_3$ — weights of $S_3$ is given by $W = \{27, 28, \ldots , 54\}$. Now, the $P_3$ — weight 27 is getting exactly one possible 5 elements sum (3, 1, 2, 10, 11) and hence the label of edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ is $f(e_1) = 10$ and $f(e_2) = 11$. Since the minimum possible sum of vertices labels for $P_3$ — weight is 8, it follows that there is no $P_3$ — weight 29, which is a contradiction. A similar contradiction arises, if the edges $e_1 = v_0v_1$ and $e_2 = v_0v_2$ with $f(e_1) = 11$ and $f(e_2) = 10$ for the $P_3$ — weight 27 is used to getting the $P_3$ — weight 29.

**Case (ii)** $f(v_0) = 7$ Then $a = 30$. Now, if $f$ is a super $(30, 1) - P_3$-antimagic total labeling of $S_9$, then by Theorem 1 [3], $\tilde{f}$ is a super $(27, 1) - P_3$-antimagic total labeling, which does not exist by Case (i).

**Theorem 7.** The star $S_9$ has no super $(a, 1) - P_3$-antimagic total labeling.

**Proof.** Let $V(S_9) = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ be the vertex set of star $S_9$. Suppose there exists a super $(a, 1) - P_3$-antimagic total labeling $f : V \cup E \rightarrow \{1, 2, 3, \ldots , 19\}$ for $S_9$ and let $v_0$ be the central vertex of $S_9$. In the computation of $P_3$ — weights the label of the central vertex $v_0$, $f(v_0)$ is used 36 times and label of other vertices and edges say $i$ are used 8 times each. Therefore,

$$28f(v_0) + 8 \sum_{i=1}^{19}(i) = \frac{36}{2}[2a + 35],$$

which implies that we get

$$a = \frac{14f(v_0) + 445}{18}.$$ 

Since $1 \leq f(v_0) \leq 10$, we have that $a$ is not an integer, which is a contradiction.

From Theorem 2-3 [5], Theorem 4-7, we get the following result.

**Theorem 8.** The star $S_n$, $n \geq 3$ admits a super $(a, 1) - P_3$-antimagic total labeling if and only if $n = 3$ and 4.

**3. Conclusion and Scope**

In [5], they investigated the existence of super $(a, d)$-P3-antimagic total labeling of star $S_n$ and posed the Problem 1 [5]. This paper proved the star $S_n$ has no super $(a, 1)$-P3-antimagic total labeling, where $n = 6, 7, 8, 9$. Therefore, we have entirely solved Problem 1 [5].

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